Consider the simple radiation boundary condition

\[ \frac{\partial \phi}{\partial t} + c \frac{\partial \phi}{\partial x} = 0, \]

where \( c \) is constant.

a. Write down the forward time/backward space finite-difference form of the boundary condition;

b. For a wave of frequency \( \omega \) and wave number \( k \) incident upon the open boundary at \( x = 0 \), find the reflection coefficient for the boundary condition;

c. Show that for \( c = 0 \) and \( c = \infty \), the boundary is a perfect reflector;

d. Show (graphically or other ways) that the magnitude of the reflection coefficient is a minimum when \( \omega / k = c \).