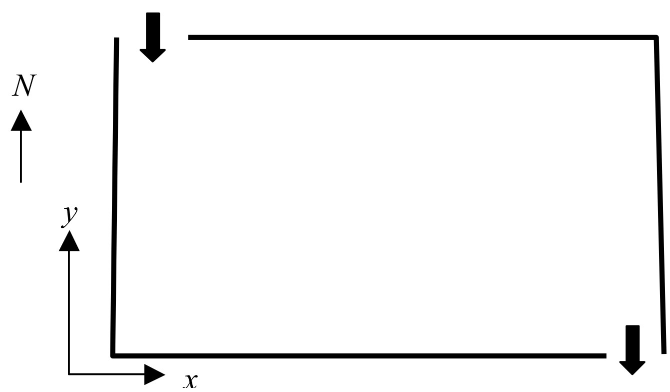
MODELLING PROBLEMS #4

Consider the steady circulation of a barotropic ocean in a rectangular basin on a β-plane with sloping bottom and bottom friction. An inflow occurs at the western edge of the northern boundary, while an outflow occurs at the eastern edge of the southern boundary. The flow is described by the streamfunction equation:

$$\psi_{xx} + \psi_{yy} + \left(\frac{\beta h_o}{r} - \frac{f_o \alpha}{r}\right)\psi_x - \frac{2\alpha}{h_o}\psi_y = 0$$

where x is the east, y north, r the constant bottom friction parameter, h_o the mean depth, f_o the mean Coriolis parameter, β the latitudinal variation in f, and α the north-south bottom slope. The streamfunction is defined by $uh_o = \psi_y$ and $vh_o = -\psi_x$ where u is the east velocity and v the north velocity.



The parameters are $f = 10^{-4}$ sec^{-1}, $h_o = 3x10^3$m, r = 10^{-3} sec^{-1}. The basin is 100 km by 100 km, and the inflow and outflow regions are 5 km wide. Using a grid spacing of Δx = Δy = 5 km, solve for ψ in the following cases:

a) $\beta = 0, \alpha = 0$. Find ψ using Jacobi iteration, Gauss-Seidel iteration and SOR. Compare convergence rates for the scheme. Find the optimal ω for SOR and verify that is works best.

b) $\beta = 2\times10^{-11} m^{-1}s^{-1}, \alpha = 10^{-4}, 6\times10^{-4}, 10^{-3}$. Find ψ using SOR with ω = 1.7. Interpret the results physically. How do the solutions change if the flow direction is reversed?

Note: The objective of giving this modeling problem is to test if students could learn numerical methods by themselves. To do this problems, you should go to read some literatures about Jacobi iteraction, Gauss-Seidel iteration and SOR.