Friction and Ocean Turbulence Part I

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<u>3 Types of Flow</u>

- **<u>Potential Flow</u> No friction; no vorticity**
- **<u>Laminar Flow</u> Friction but no overturning**
- **<u>Turbulent Flow</u> Friction, high vorticity, overturning**

The Road to Turbulence

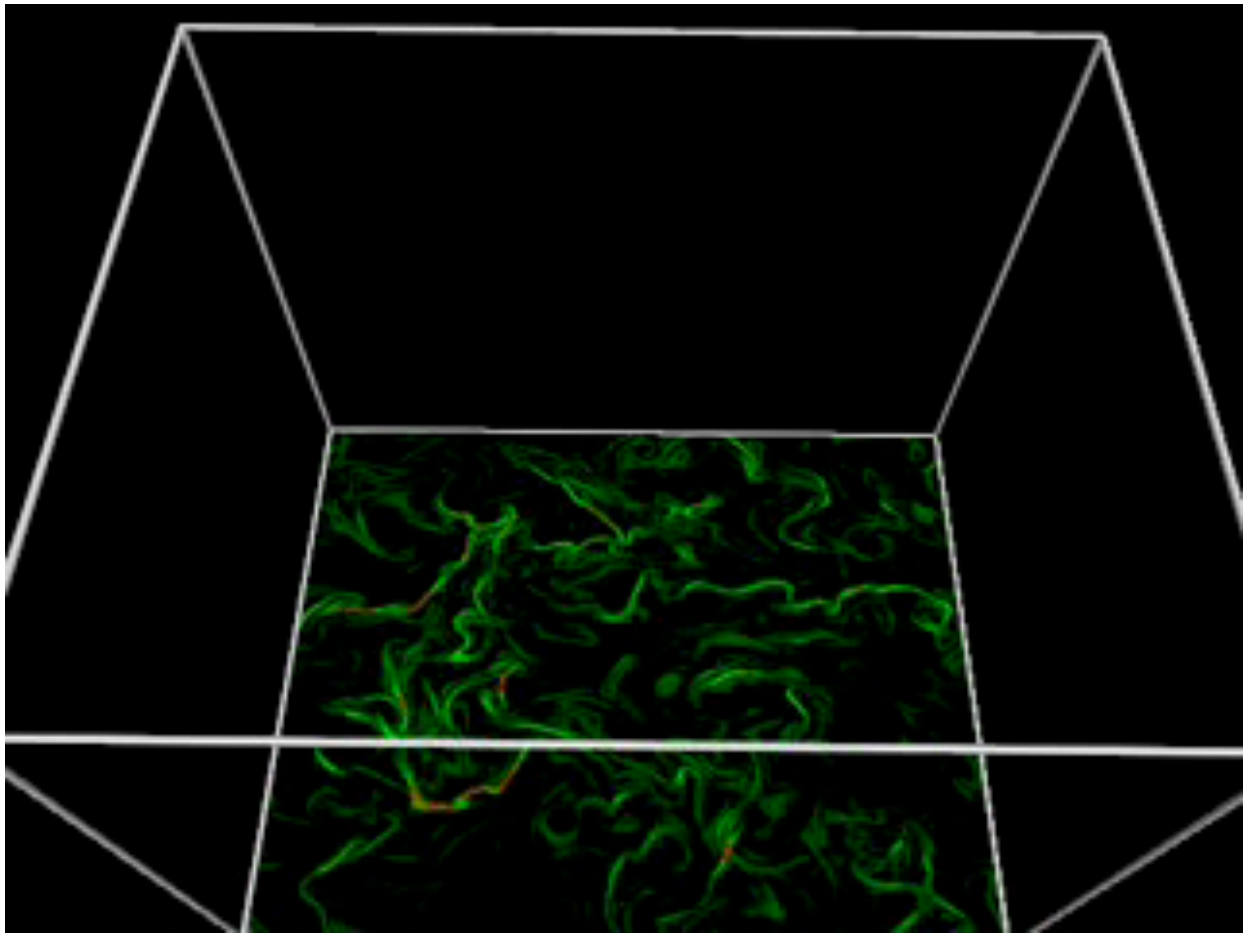
Turbulent Flow

Laminar flow

Who is this?

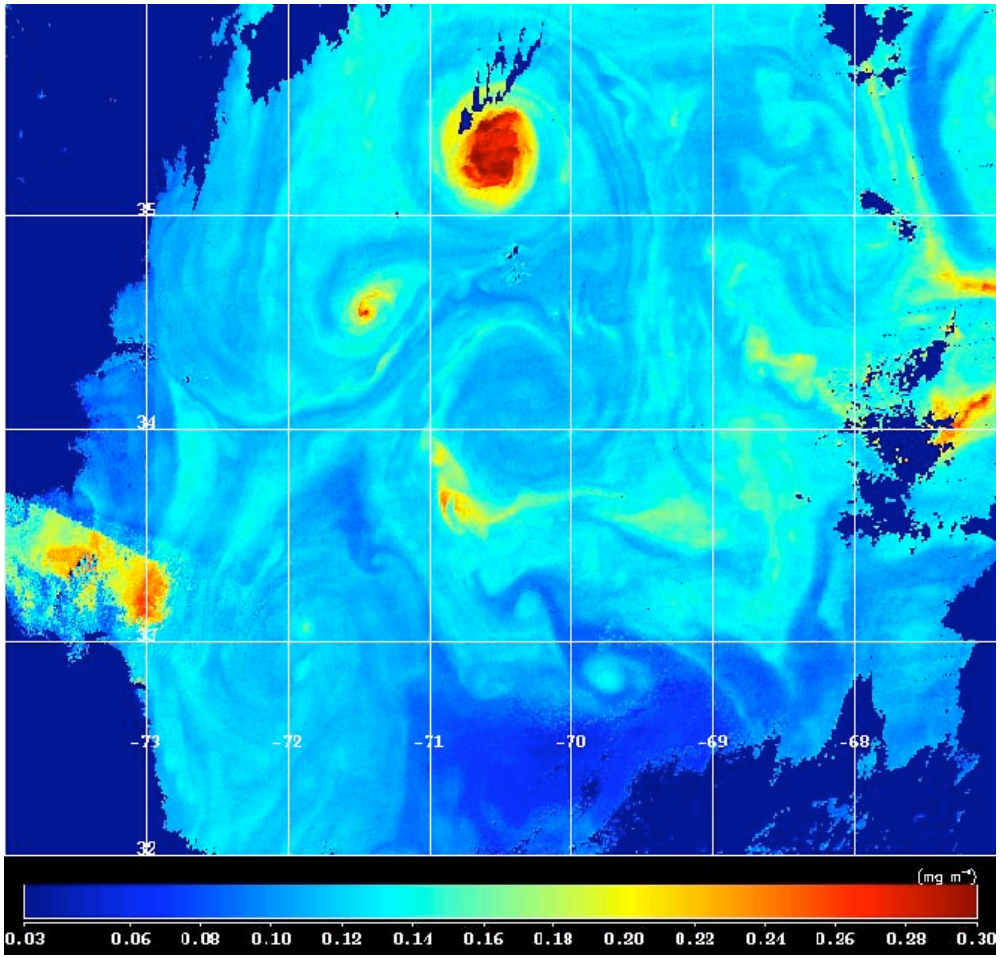
Homework problem: Give one example of ocean flows which are: (1) potential flow; (2) laminar flow;(3) turbulent flow

Numerical Model of 3 D Turbulence



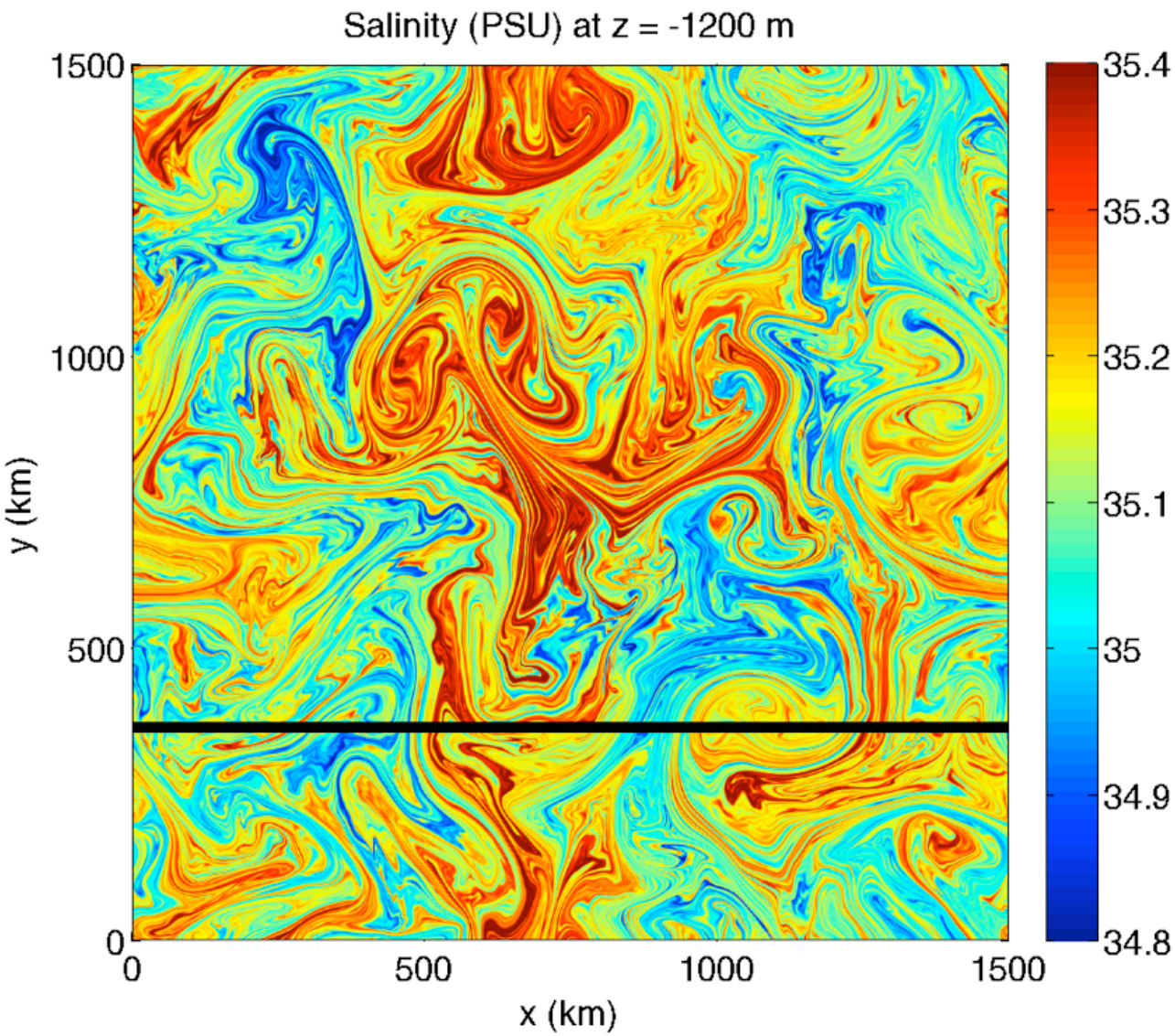
Direct Numerical Simulation (DNS) model of turbulence

2 D Ocean Turbulence

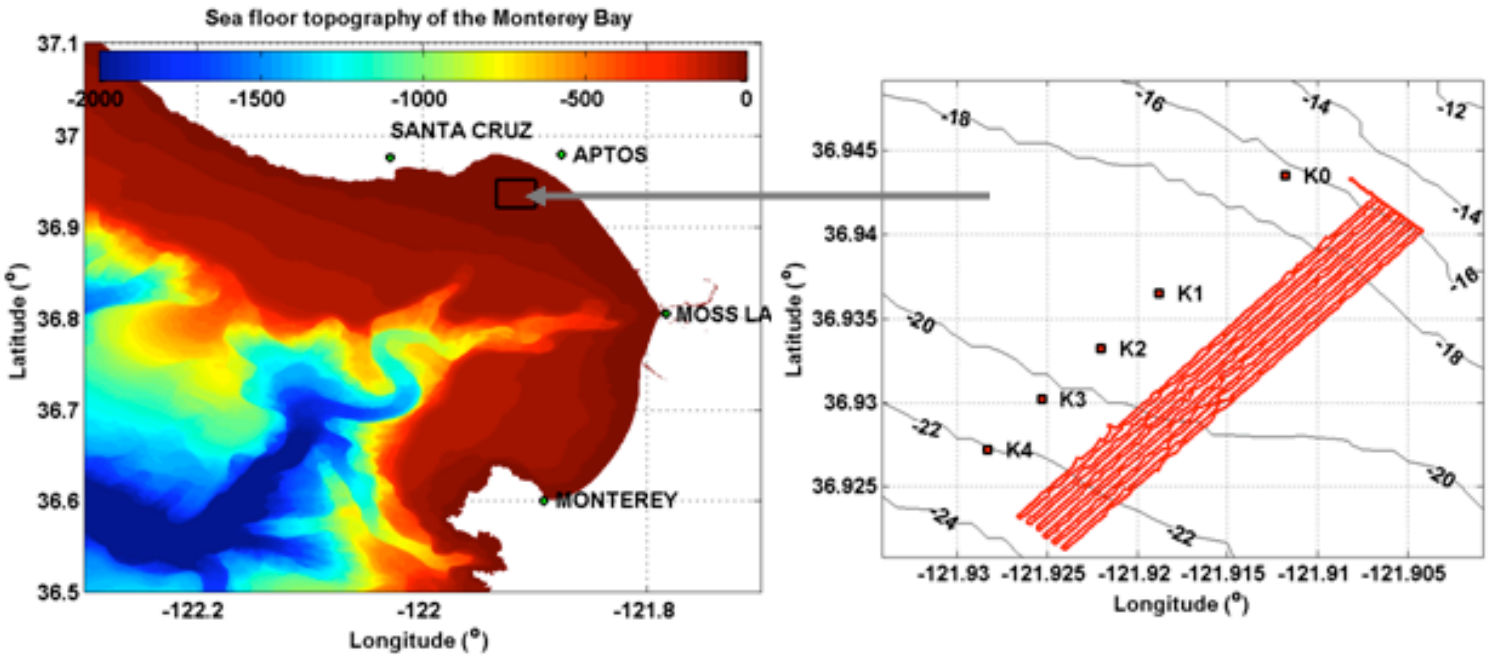


Sea surface chlorophyll distribution derived from sea surface color in the western Sargasso Sea on May 27, 2007

High Resolution Numerical model of a Frontal Instability Producing 2 D turbulence



How do we measure ocean 3D turbulence?

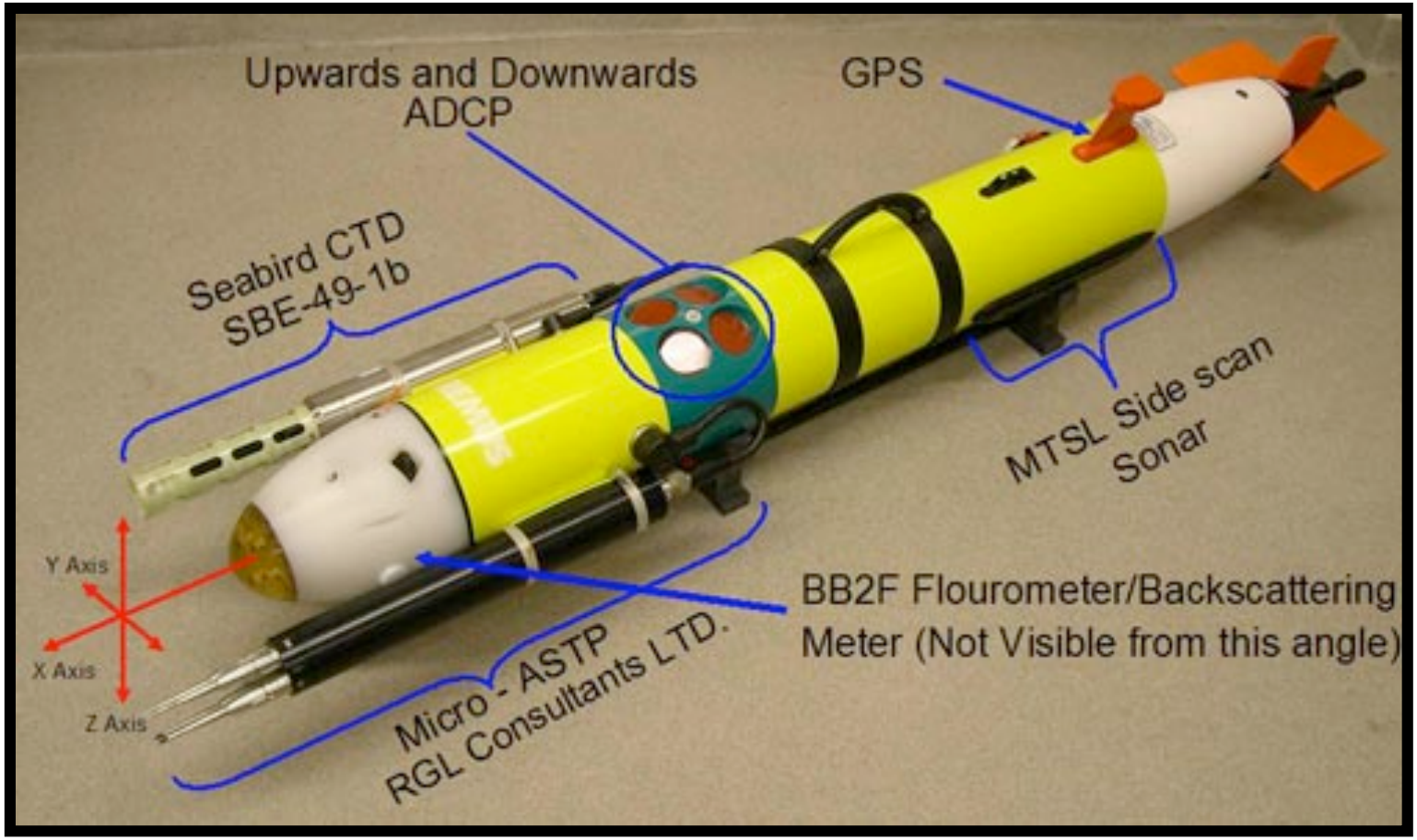


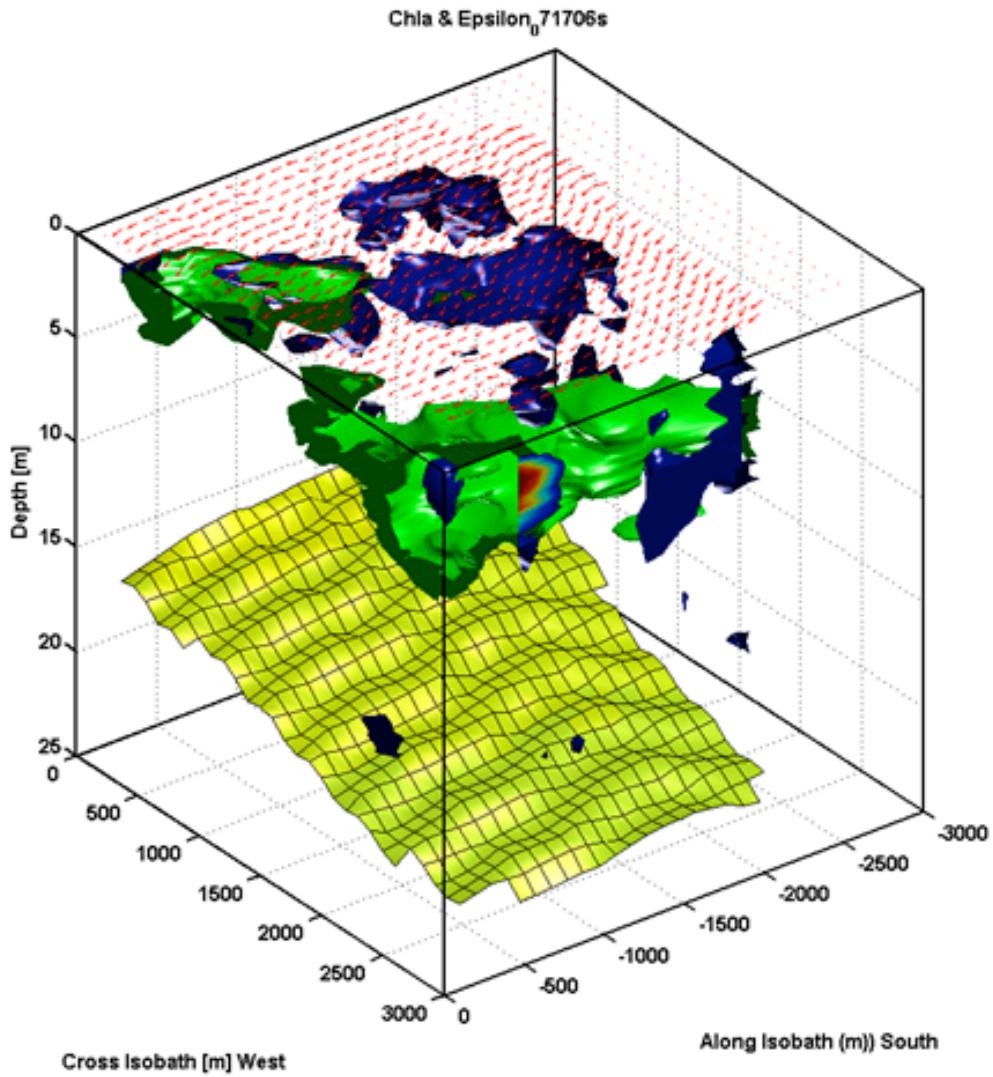
Layered Organization of the Coastal Ocean (LOCO)
Experimental Site Monterey Bay CA

SMAST LOCO Objective: Study relationship of biological thin layers to turbulence

SMAST T-REMUS Autonomous Underwater Vehicle

Used for ocean turbulence measurements

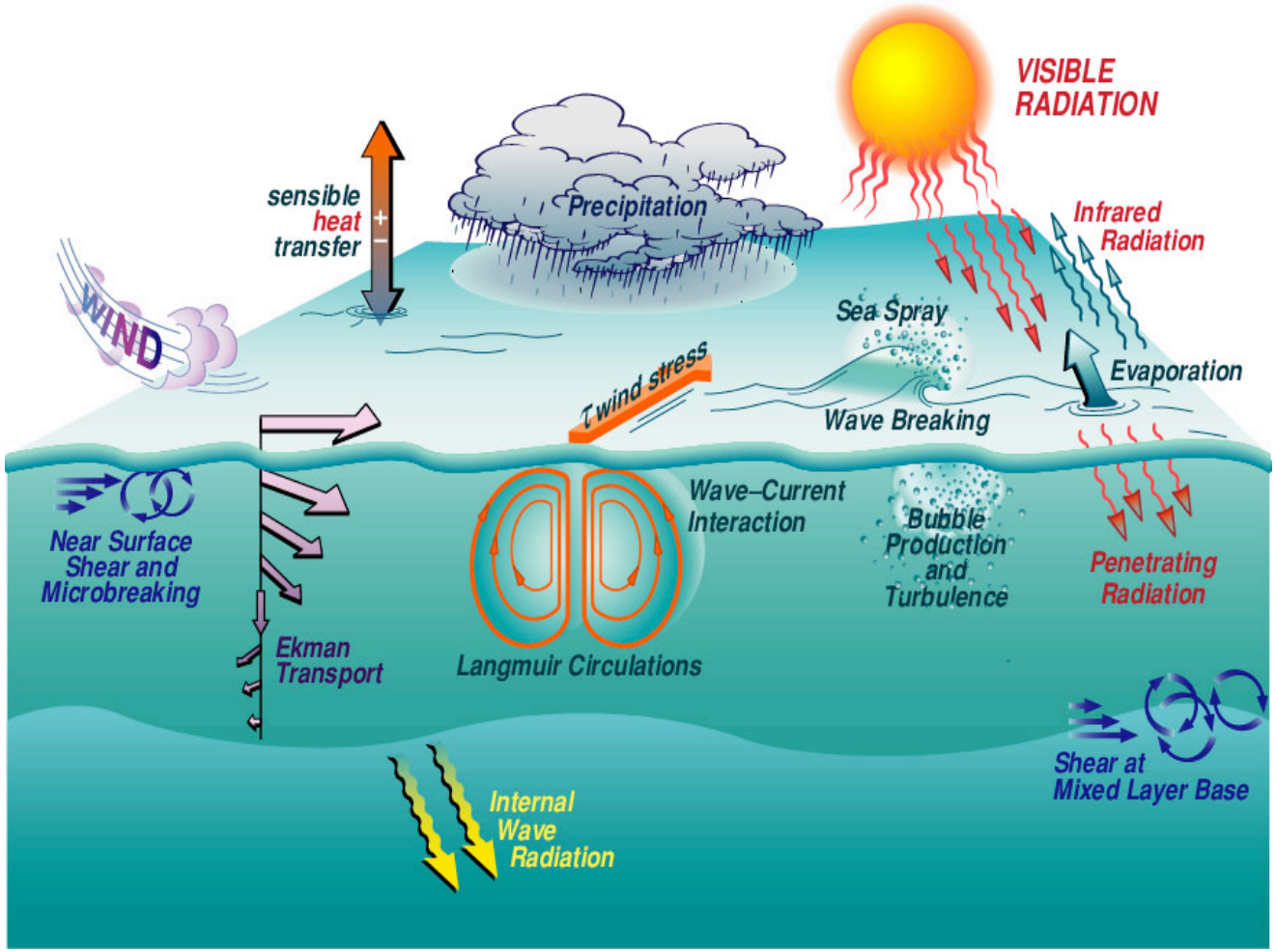




Green chlorophyll; Blue turbulent dissipation rate

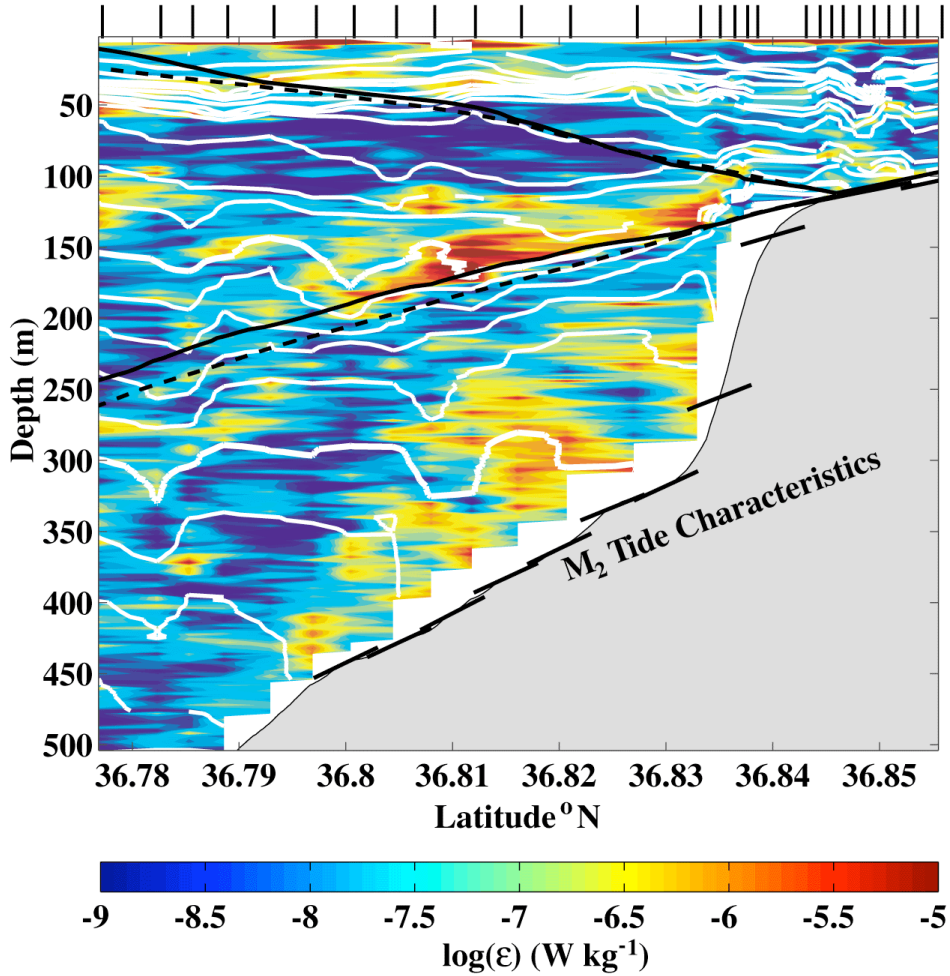
Why is turbulence important in oceanography?

I. Air Sea Interaction



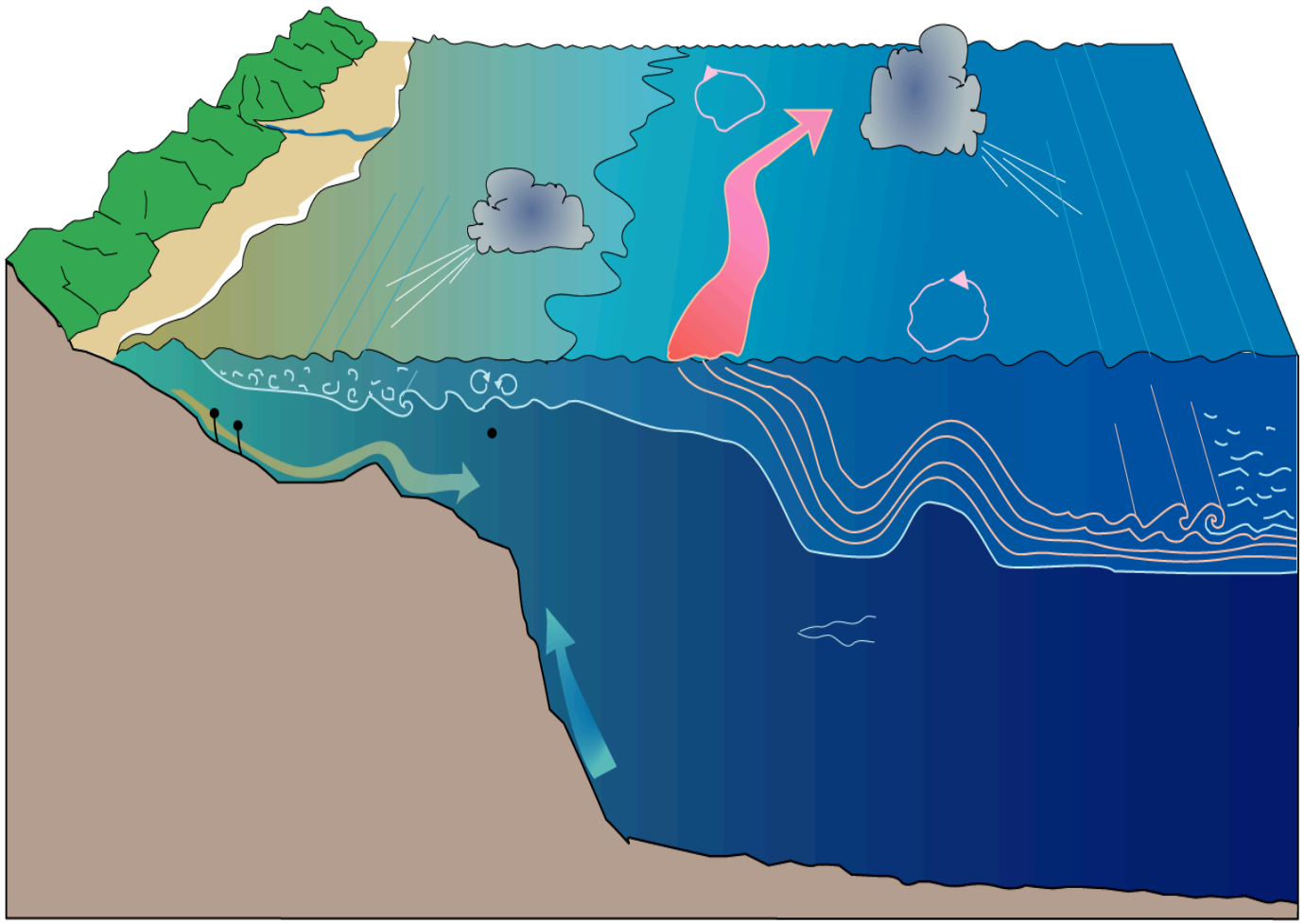
II. Interior Mixing

(temperature (heat) salt, momentum, PV, nutrients, plankton, larvae, bubbles, other constituents)

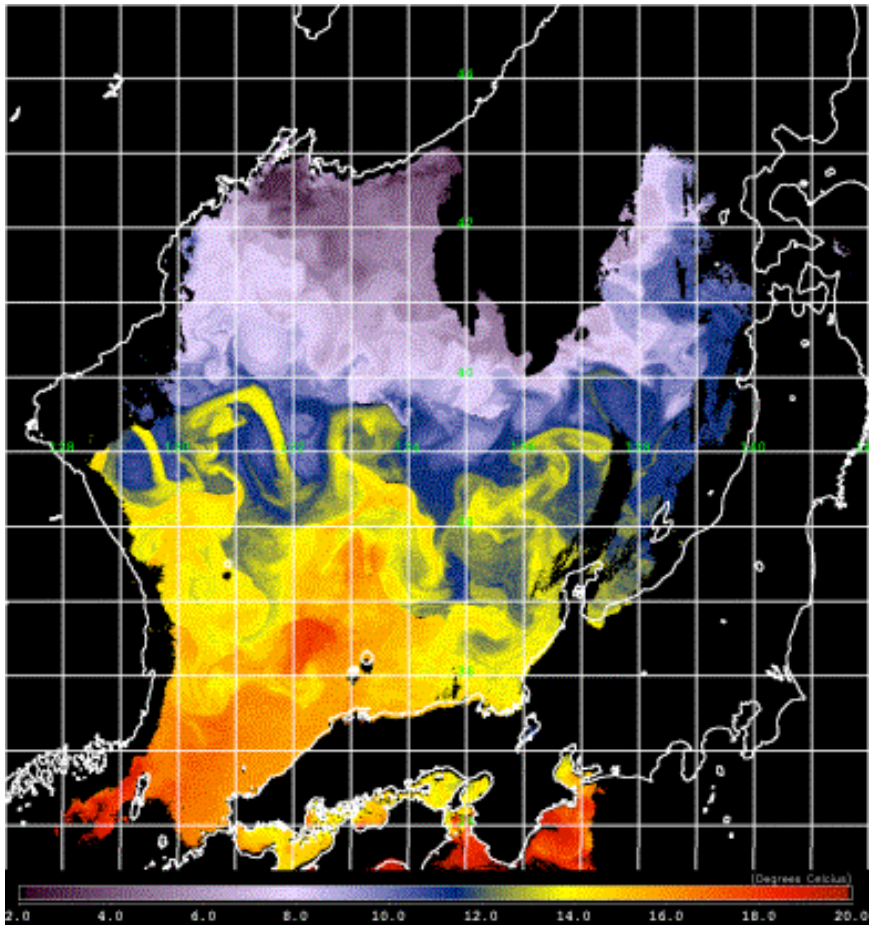


Turbulence produced by an underwater Internal wave beam
Courtesy of M. Gregg

III. Boundary Friction (Side and bottom)



IV. Eddy (2 D Turbulent) Mixing



False Color Image of temperature in Sea of Japan

Turbulent Frictional Effects: The Vertical Reynolds Stress

No Turbulence

$$\frac{du_i}{dt} + (\vec{f} \times \vec{u})_i = -\frac{\partial p}{\partial x_i} - g_i + \cancel{Fr_i}$$

or in component form

$$\frac{du}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) + fv + \cancel{Fr_x}$$

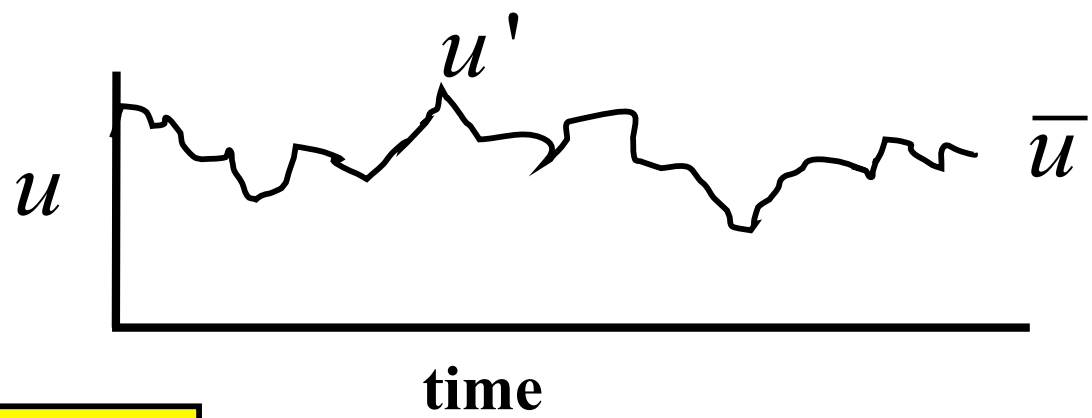
$$\frac{dv}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) - fu + \cancel{Fr_y}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) - g + \cancel{Fr_z}$$

Mean and Fluctuating Quantities

$$u = \bar{u} + u'$$

mean + fluctuating



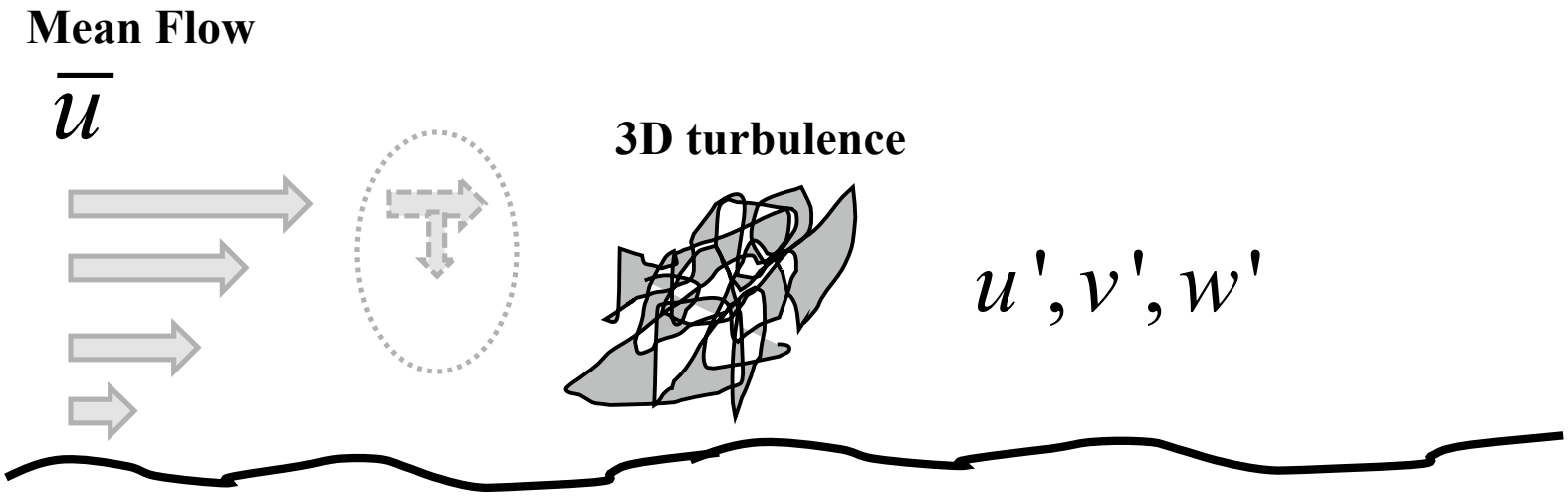
Three Types of Averages

- **Ensemble**
- **Time**
- **Space**

Ergodic Hypothesis: Replace ensemble average by either a space or time average

Notation $\bar{q} \equiv \langle q \rangle$

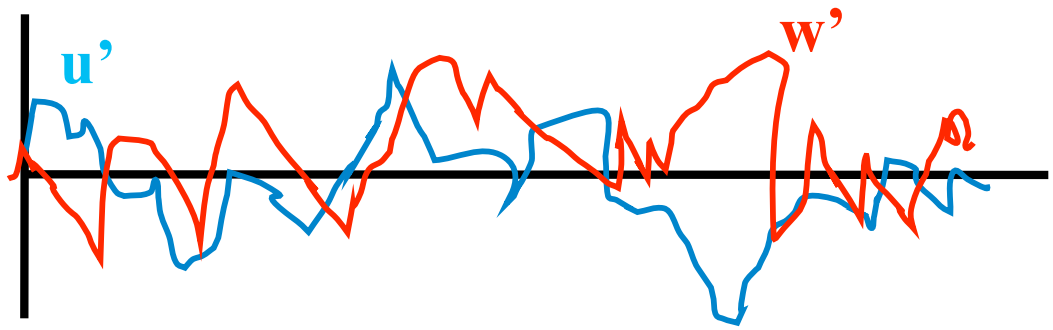
How does the turbulence affect the mean flow?



Concept of Reynolds Stress

$\tau = -\rho < u'w' >$

$$< u'w' > \quad < 0$$



Momentum Equation with Molecular Friction

$$\frac{du}{dt} - fv = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) + Fr_x$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) + Fr_y$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) - g + Fr_z$$

where

$$\frac{d}{dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

But

$Fr_{x,y,z} \propto \nu \times$(derivative of velocity shear)

ν = molecular viscoisty = $10^{-6}\ \frac{m^2}{\sec}$

$$\frac{du}{dt} - fv = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) - g$$

where

$$\frac{d}{dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

But $u_i = \bar{u}_i + u'_i$

$u_x = \bar{u} + u'$

$u_y = \bar{v} + v'$

$u_w = \bar{w} + w'$

Example
Uniform unidirectional wind blowing over ocean surface

Dimensional Analysis

Boundary Layer Flow

•Gradient in "x" direction smaller than in "z" direction

•*Mean velocity unidirectional , no gradient in "y" direction*

$$\frac{du}{dt} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) - g$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \mathrm{v}\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

using $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v - \frac{\partial}{\partial z}w = 0$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{uu}{2}\right) + \cancel{\frac{\partial(vu)}{\partial y}} - \frac{\partial(wu)}{\partial z}) - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial (uw)}{\partial z} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

But $\mathrm{u_i} = \overline{\mathrm{u}}_\mathrm{i} + \mathrm{u'_i}$

$\mathrm{u_x} = \overline{u} + u'$

$\mathrm{u_y} = \overline{\mathrm{v}} + \mathrm{v'}$

$\mathrm{u_w} = \overline{w} + \mathrm{w'}$

$$(\overline{u} + \cancel{u'})\frac{\partial(\overline{u} + \cancel{u'})}{\partial x} + \frac{\partial(\overline{u} + u')(\overline{w} + \mathrm{w'})}{\partial z}$$

Now we average the momentum equation

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \frac{\partial(<u'w'>)}{\partial z} - f\overline{\mathrm{v}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x}$$

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} - f\overline{\mathrm{v}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

where

$\tau_x = -\rho < u'w' > =$ "x" component of Reynolds Stress

General Case of Vertical Turbulent Friction

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_i} + (\vec{f} \times \vec{\overline{u}})_i = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_i}{\partial z} - g_i$$

where

$$\tau_{\mathrm{i}} = -\rho < u_i' w' > \quad \text{for } u'_x = u', u'_y = \mathrm{v}'$$

$$= 0 \qquad \mathrm{i} = 3 \ (\mathrm{z})$$

Note that we sometimes use 1,2,3 in place x, y, z as subscripts

Convention: When we deal with typical mean equations we drop the "mean" Notation!

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + (\vec{f} \times \vec{u})_i = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_i}{\partial z} - g_i$$

Component form of Equations of Motion with Turbulent Vertical Friction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-f\mathrm{v}=-\frac{1}{\rho}(\frac{\partial p}{\partial x})+\frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$\frac{\partial \mathrm{v}}{dt}+u\frac{\partial \mathrm{v}}{\partial x}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial y}+w\frac{\partial \mathrm{v}}{\partial z}+fu=-\frac{1}{\rho}(\frac{\partial p}{\partial y})+\frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+\mathrm{v}\frac{\partial \mathrm{w}}{\partial y}+w\frac{\partial \mathrm{w}}{\partial z}=-\frac{1}{\rho}(\frac{\partial p}{\partial z})-g$$

Note: in many cases the mean vertical velocity is small and we can assume w = 0 which leads to the hydrostatic approximation and

$$(1)\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}-f\mathrm{v}=-\frac{1}{\rho}(\frac{\partial p}{\partial x})+\frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$(2)\frac{\partial \mathrm{v}}{dt}+u\frac{\partial \mathrm{v}}{\partial x}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial y}+fu=-\frac{1}{\rho}(\frac{\partial p}{\partial y})+\frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

$$(3)\quad \frac{\partial p}{\partial z}=-\rho g$$