Geostrophy

L. Goodman

General Physical Oceanography
MAR 555

School for Marine Sciences and Technology
Umass-Dartmouth
Geostrophy  IPO-10

Page 65 Ocean Circulation
Horizontal Equation of Motion

\[
\frac{D\vec{u}}{Dt} + f \times \vec{u} = -\frac{1}{\rho} \nabla_h p + \frac{1}{\rho} \frac{\partial}{\partial z} \vec{\tau}
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \quad \vec{\tau} = -\langle \vec{u}' \vec{w}' \rangle = k \frac{\partial \vec{u}}{\partial z}
\]

\[
\frac{\partial u_i}{\partial t} + u \frac{\partial u_i}{\partial x} + v \frac{\partial u_i}{\partial y} + w \frac{\partial u_i}{\partial z} + (f \times \vec{u})_i = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + k \frac{\partial^2 u_i}{\partial z^2} + A_h \left( \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} \right)
\]
Road to Geostrophy: Dimensional Analysis

**Acceleration**
\[
\frac{\partial u_i}{\partial t} + u \frac{\partial u_i}{\partial x} + v \frac{\partial u_i}{\partial y} + w \frac{\partial u_i}{\partial z} = fU \quad (\hat{f} \times \vec{u})_i
\]

**Advection**
\[
U \frac{U^2}{T} = \frac{U^2}{L} \quad \frac{WU}{H} = \frac{U^2}{L}
\]

**Coriolis**
\[
-\frac{1}{\rho_0} \frac{\partial p}{\partial x} + k \frac{\partial^2 u_i}{\partial z^2}
\]

**Pressure Gradient**

**Vertical Friction**
\[
A_h \left( \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} \right)
\]

**Horizontal Friction**

\[
Ro = \frac{U}{fL} \quad \text{Rossby Number}
\]

If \( Ro \ll 1, E_z \ll 1, E_h \ll 1 \)

Geostrophy \( (\hat{f} \times \vec{u})_i = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} \)
Homework: The mesoscale or eddy scale is typically from 20 to 200 km with characteristic velocities of order .5 m/sec for the energetic eddies. Do you still expect geostrophy to hold? When would you expect it to least hold and what term in the horizontal equation of motion would you need to consider?

Open Ocean Circulation Characteristic Scales

$L \sim 10^6 \text{m} \quad H \sim 10^3 \text{m}$

$f = 10^{-4} \text{sec} \quad U \sim 10^{-1} \frac{m}{\text{sec}}$

$k = 10^{-1} - 10^{-4} \frac{m^2}{\text{sec}} \quad A = 10 - 10^5 \frac{m^2}{\text{sec}}$

$Ro = \frac{U}{fL} = 10^{-3}$

$E_z = \frac{k}{fH^2} = (10^{-3} - 10^{-6})$

$E_h = \frac{A}{fL^2} = (10^{-3} - 10^{-7})$
Geostrophic Equations

\[ \frac{\partial p}{\partial x} = \rho f v \]
\[ \frac{\partial p}{\partial y} = -\rho f u \]

Hydrostatic Equation

\[ \frac{\partial p}{\partial z} = -\rho g \]

Barotropic Flow \( \rho = \rho_0 \)

\[ \alpha = \frac{\partial \zeta}{\partial x} > 0 \]

\[ p = \rho_0 g (z + \zeta) \]

\[ \frac{\partial p}{\partial x} = \rho_0 g \alpha \]

\[ v = \frac{1}{\rho_0 f} \frac{\partial p}{\partial x} = \frac{g \alpha}{f} \]

“y” direction

- Flow into screen
- Flow out screen

\( p_A > p_B \)
Homework: Given an west–east the sea surface height profile shown below using geostrophy make a sketch of magnitude and “x” position of what you think the “y” direction surface velocity profile will look like.
Homework Question: IPO figure 10.3. Topex/Poseidon altimeter observations of the Sea Surface Height (SSH) of the Gulf Stream region. Using geostrophy what can you say about the direction of flow of: (a) the Gulf Stream; (b) the northern and southern sides of the warm core rings; (c) the northern and southern sides of the cold core rings?
**Example: Two Layer Geostrophic Flow**

Surface slope \( \alpha = \frac{h}{L} \)

Density slope \( \gamma = \frac{H}{L} \)

\[ p_A = \rho_1 g (z + h) \]
\[ p_B = \rho_1 g (z - H) + \rho_2 g H \]
\[ \frac{\partial p}{\partial x} = \frac{p_A - p_B}{L} = \frac{\rho_1 g h - (\rho_2 - \rho_1) g H}{L} \]
\[ \frac{\partial p}{\partial x} = \rho_1 g \alpha - \Delta \rho g \gamma \Rightarrow v_2 = g \frac{\alpha}{f} - g' \frac{\gamma}{f} = v_1 - v_r \]

\( v_1 = \frac{g \alpha}{f} \)

\( v_r = \frac{g'}{f} \gamma \)

where \( g' = \frac{\ddot{\alpha}}{\ddot{n}} g \)

reduced gravity
Concept of Level of No Motion

\[ v_1 = \frac{g \alpha}{f} \]

\[ v_2 = 0 \]

\[ v_2 = v_1 - v_r = v_1 - g' \frac{\gamma}{f} \]

if \( v_2 = 0 \) \( \Rightarrow v_1 = g' \frac{\gamma}{f} = v_r \)
General Baroclinic Flow \( \rho = \rho(x, z) \)

Sea surface slope \( \alpha > 0 \)

\[ \begin{align*}
\text{Surface velocity} & \quad \mathbf{v}_s = \frac{1}{\rho_1 f} \frac{\partial p}{\partial x} = \frac{g'\gamma}{f} \\
\text{v}_r & = \frac{1}{\rho_1 f} \frac{\partial p}{\partial x} = \frac{g'\gamma}{f}
\end{align*} \]

\( \gamma = \gamma(x, z) \) slope of density surface

Homework: Often it is very difficult to measure the sea surface slope for large scale flow. But it is often a good assumption to use geostrophy and the concept of a level of no motion. Explain how you could use a CTD to find surface currents.
Geostrophic Shear

\[ \frac{\partial}{\partial z} \frac{\partial p}{\partial x} = \frac{\partial}{\partial z} (\rho f v) = f v \frac{\partial p}{\partial z} + f \rho \frac{\partial v}{\partial z} \]

\[ \frac{\partial}{\partial z} \frac{\partial p}{\partial y} = -\frac{\partial}{\partial z} (\rho f u) = f u \frac{\partial p}{\partial z} + f \rho \frac{\partial u}{\partial z} \]

\[ \frac{\partial}{\partial x} \{ \frac{\partial p}{\partial z} \} = -g \frac{\partial \rho}{\partial x} \quad \frac{\partial}{\partial y} \{ \frac{\partial p}{\partial z} \} = -g \frac{\partial \rho}{\partial y} \]

"Thermal" Wind Equations

\[ \Rightarrow \frac{\partial v}{\partial z} = \frac{g}{f \rho} \frac{\partial \rho}{\partial x} \quad \frac{\partial u}{\partial z} = -\frac{g}{f \rho} \frac{\partial \rho}{\partial y} \]

Note: \[ \frac{\delta v}{\delta z} \approx \frac{g}{f \rho} \frac{\delta \rho}{\delta x} \Rightarrow \delta v \approx v_r = \frac{1}{f} g \frac{\delta \rho}{\rho \delta x} \frac{\delta z}{\delta x} = \frac{g' \gamma}{f} \]
Sea Surface Slope $\alpha$

Isopycnal Slope $\gamma(x, z)$

Blue Lines Isopycnals

$v_s = \frac{g\alpha}{f}$

$\gamma'(x, z)$

$\gamma(x, z)$

$V = V_s - V_r$
Older Traditional Approach to Estimating Surface Currents.

IPO figure 2.8. The time-averaged, surface circulation of the ocean deduced from nearly a century of oceanographic expeditions. From Tolmazin (1985). An assumption of a depth of no motion is made to obtain the surface velocity estimates from geostrophy.
Modern approach to estimating surface currents

IPO Figure 10.5 Global, time-averaged topography of the ocean for the period 1992–2002 from a joint analysis of drifter, satellite altimeter, wind, and the GRACE Gravity Model-01 data by Nikolai Maximenko (IPRC) and Peter Niiler (SIO).

Geostrophic currents at the ocean surface are parallel to the contours.
Homework: Using the concept of geostrophy and a level of no motion at $z = 600\text{m}$ to estimate the surface current of the Gulf Stream. Use the dotted section for your calculation.
Conservation of Potential Vorticity and Deep Circulation

\[ \frac{\partial p}{\partial x} = \rho f v \]
\[ \frac{\partial}{\partial y} \left( \frac{\partial p}{\partial x} - \frac{\partial p}{\partial y} \right) = 0 \]
\[ \frac{\partial p}{\partial y} = -\rho f u \quad \Rightarrow \quad \rho \beta v + \rho f (\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}) \]

using \[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]
\[ \Rightarrow \beta \frac{\partial w}{\partial z} \]
\[ w = \frac{\beta}{f} \int_0^H vdz = \frac{\beta}{f} V \]

or \[ V = \frac{fw_H}{\beta} \]

V vertical integral of northward velocity

H: height of some “layer” of water
Why does the deep interior flow go equatorward in both hemispheres?

Answer: \( V = \frac{f}{\beta} W_H \). Explain.