Geostrophy

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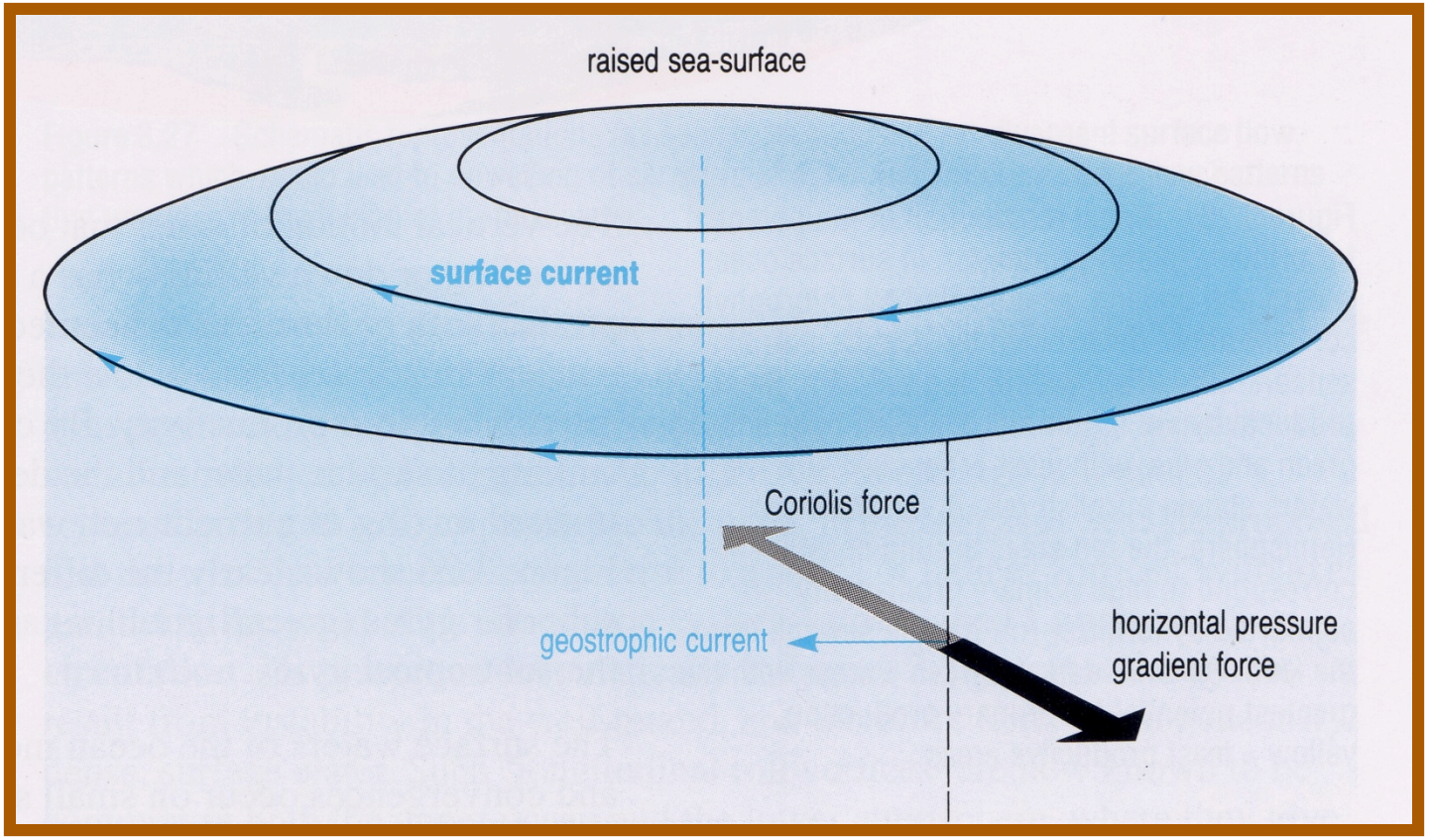
Geostrophy IPO-10



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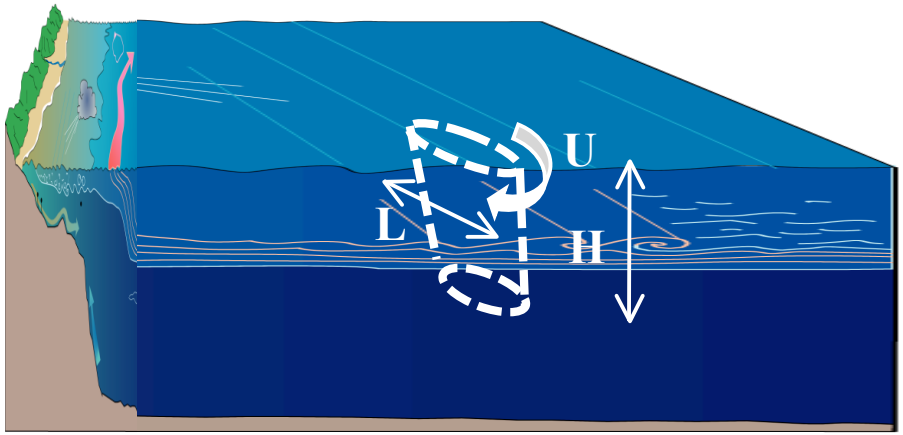
Geostrophy IPO-10



Horizontal Equation of Motion

$$\frac{D\vec{u}}{Dt} + \vec{f} \times \vec{u} = -\frac{1}{\rho}\nabla_h p + \frac{1}{\rho}\frac{\partial}{\partial z}\vec{\tau}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \qquad \frac{\vec{\tau}}{\rho} = -<\vec{u}'w'> = k\frac{\partial \vec{u}}{\partial z}$$

Added horizontal Friction (eddies)

$$\frac{\partial u_i}{\partial t} + u\frac{\partial u_i}{\partial x} + \mathrm{v}\frac{\partial u_i}{\partial y} + w\frac{\partial u_i}{\partial z} + (\vec{f} \times \vec{u})_i = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + k\frac{\partial^2 u_i}{\partial z^2} + A_h\left(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2}\right)$$

Road to Geostrophy: Dimensional Analysis

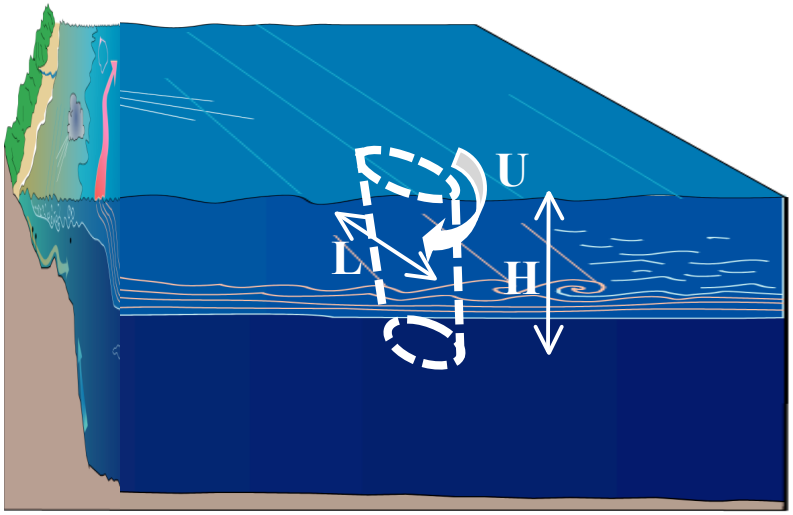
Acceleration	Advection	Coriolis		Pressure Gradient	Vertical Friction	Horizontal Friction
$\frac{\partial u_i}{\partial t}$	$+\ u\frac{\partial u_i}{\partial x}+\mathrm{v}\frac{\partial u_i}{\partial y}+w\frac{\partial u_i}{\partial z}$	$+(\vec{f}\times\vec{u})_i$	$=$	$-\frac{1}{\rho_0}\frac{\partial p}{\partial x_i}$	$+\ k\frac{\partial^2 u_i}{\partial z^2}$	$+\ A_h(\frac{\partial^2 u_i}{\partial x^2}+\frac{\partial^2 u_i}{\partial y^2})$
$\frac{U}{T}=\frac{U^2}{L}$	$\frac{U^2}{L}$	fU		?	$\frac{kU}{H^2}$	$\frac{AU}{L^2}$
$Ro=\frac{U}{fL}$ ***Rossby Number***		1		?	$E_z=\frac{k}{fH^2}$ ***Vertical EkmanNumber***	$E_h=\frac{A_h}{fL^2}$ ***Horizontal Ekman Number***

Divide by fU

If $Ro << 1, E_z << 1, E_h << 1$

Geostrophy $\quad (\vec{f}\times\vec{u})_i = -\frac{1}{\rho}\frac{\partial p}{\partial x_i}$

Open Ocean Circulation Characteristic Scales



$L \sim 10^6 m \qquad H \sim 10^3 m$

$f = 10^{-4} sec \qquad U \sim 10^{-1} \frac{m}{sec}$

$k = 10^{-1}\text{-}10^{-4} \frac{m^2}{sec} \qquad A = 10\text{-}10^5 \frac{m^2}{sec}$

$$Ro = \frac{U}{fL} = 10^{-3}$$

$$E_z = \frac{k}{fH^2} = (10^{-3} - 10^{-6})$$

$$E_h = \frac{A}{fL^2} = (10^{-3} - 10^{-7})$$

Homework: The mesoscale or eddy scale is typically from 20 to 200 km with characteristic velocities of order .5 m/sec for the energetic eddies. Do you still expect geostrophy to hold? When would you expect it to least hold and what term in the horizontal equation of motion would you need to consider?

Geostrophic Equations

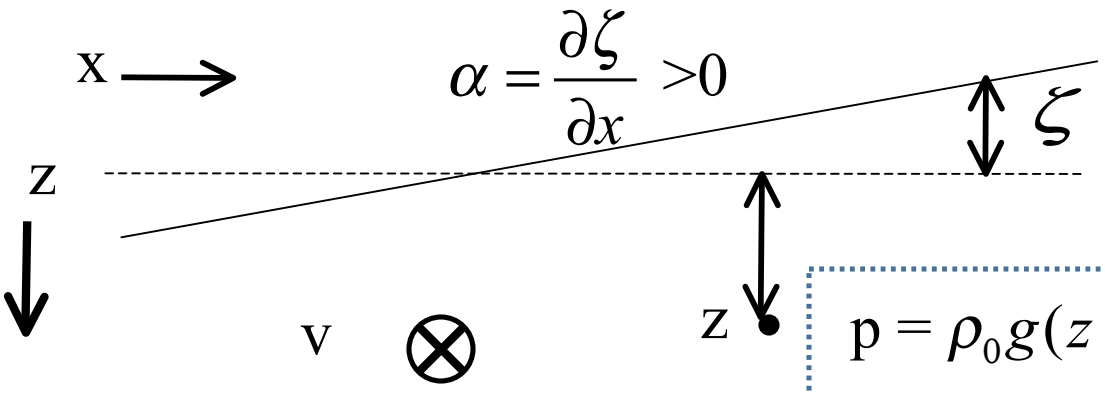
$$\frac{\partial p}{\partial x} = \rho f v$$

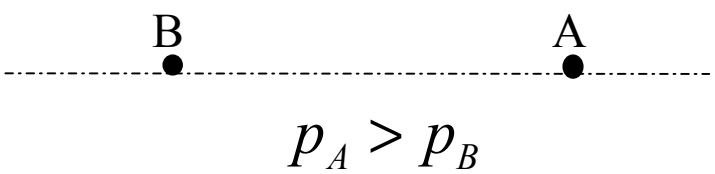
$$\frac{\partial p}{\partial y} = -\rho f u$$

Hydrostatic Equation

$$\frac{\partial p}{\partial z} = -\rho g$$

Barotropic Flow $\rho = \rho_0$





$$p = \rho_0 g(z + \zeta)$$

$$\frac{\partial p}{\partial x} = \rho_0 g \alpha$$

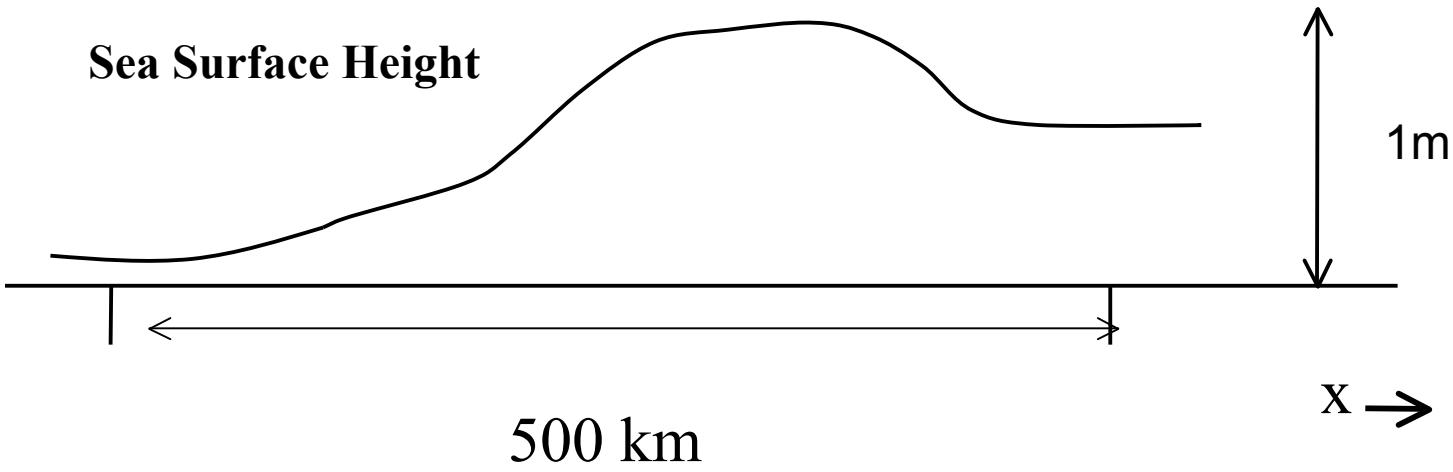
$$v = \frac{1}{\rho_0 f}\frac{\partial p}{\partial x} = \frac{g\alpha}{f}$$

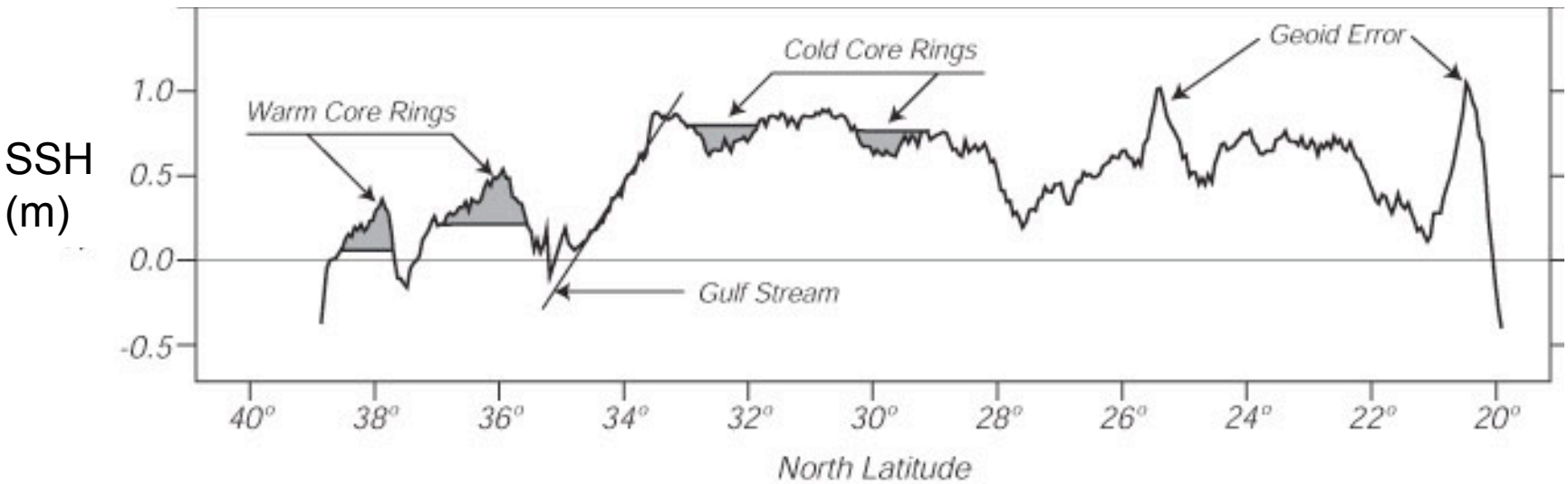
"y" direction

⊗ **Flow into screen**

⊙ **Flow out screen**

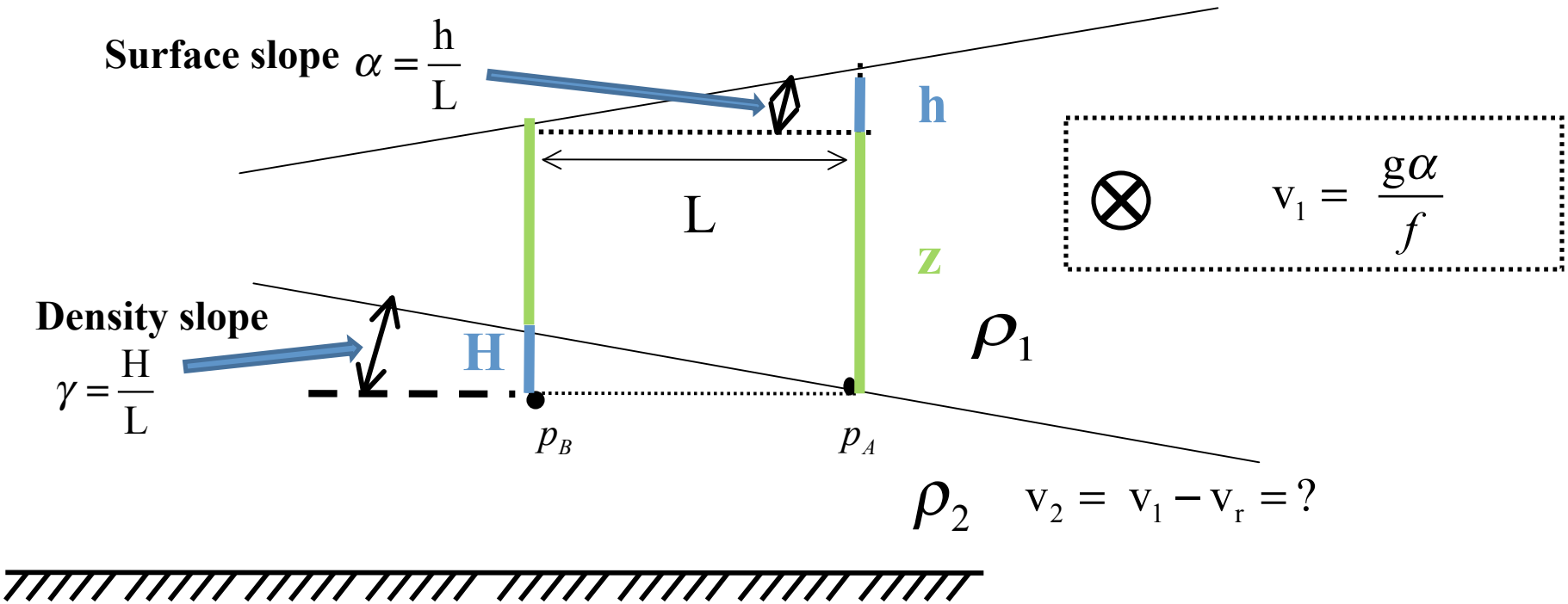
Homework: Given an west –east the sea surface height profile shown below using geostrophy make a sketch of magnitude and "x" position of what you think the "y" direction surface velocity profile will look like.





Homework Question: IPO figure 10.3. Topex/Poseidon altimeter observations of the Sea Surface Height (SSH) of the Gulf Stream region. Using geostrophy what can you say about the direction of flow of : (a) the Gulf Stream; (b) the northern and southern sides of the warm core rings (c) the northern and southern sides of the cold core rings?

Example: Two Layer Geostrophic Flow



$$p_A = \rho_1 g(z+h)$$

$$p_B = \rho_1 g(z-H) + \rho_2 gH$$

$$\frac{\partial p}{\partial x} = \frac{p_A - p_B}{L} = \frac{\rho_1 gh - (\rho_2 - \rho_1)gH}{L}$$

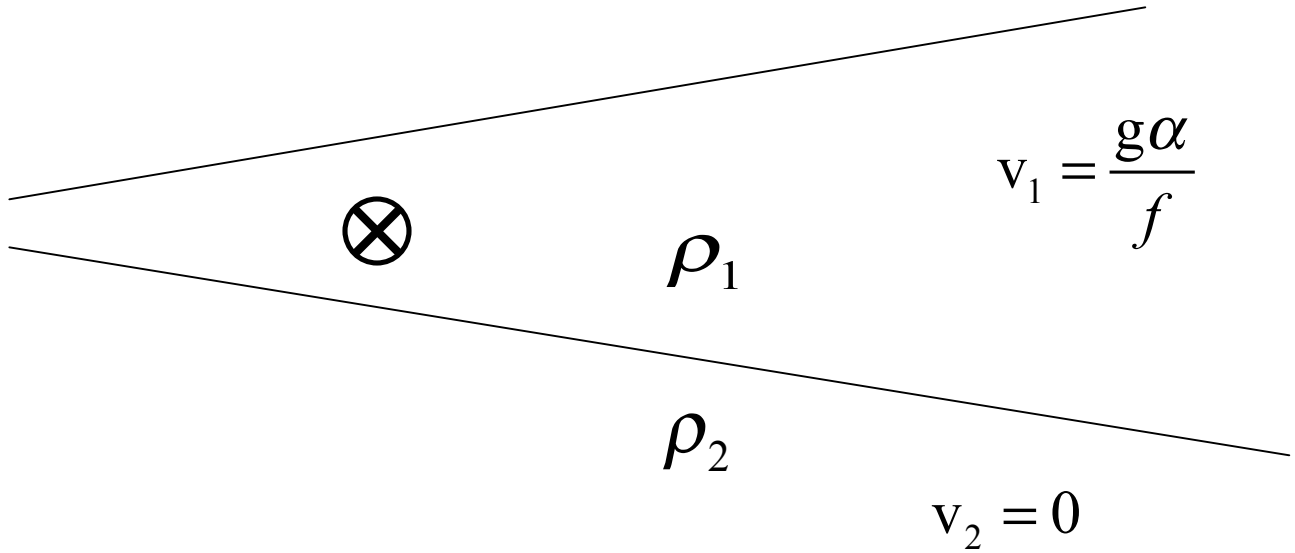
$$\frac{\partial p}{\partial x} = \rho_1 g\alpha - \Delta\rho g\gamma \Rightarrow \quad v_2 = g\frac{\alpha}{f} - g'\frac{\gamma}{f} = v_1 - v_r$$

$$v_r = g'\frac{\gamma}{f}$$

where $g' = \frac{\text{Äñ}}{\text{ñ}} g$

reduced gravity

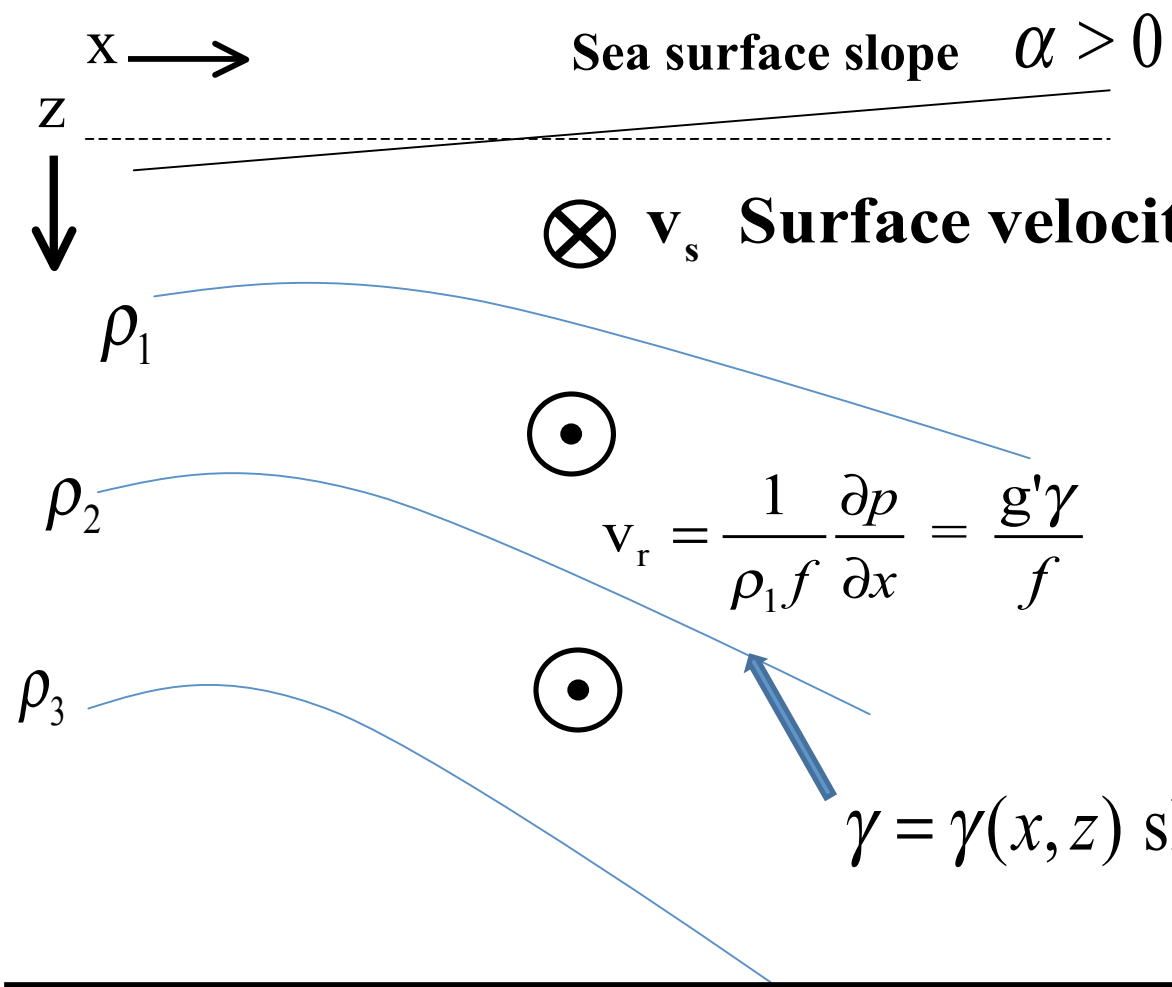
Concept of Level of No Motion



$$v_2 = v_1 - v_r = v_1 - g'\frac{\gamma}{f}$$

$$if \;\; v_2 = 0 \Rightarrow v_1 = g'\frac{\gamma}{f} = v_r$$

General Baroclinic Flow $\rho = \rho(x,z)$



Sea surface slope $\alpha > 0$

$\otimes$ v_s **Surface velocity** $v_s = \frac{1}{\rho_1 f}\frac{\partial p}{\partial x} = \frac{g\alpha}{f}$

$\odot$

$$v_r = \frac{1}{\rho_1 f}\frac{\partial p}{\partial x} = \frac{g'\gamma}{f}$$

$\odot$

$\gamma = \gamma(x,z)$ slope of density surface

Homework : Often it is very difficult to measure the sea surface slope for large scale flow. But it is often a good assumption to use geostrophy and the concept of a level of no motion. Explain how you could use a CTD to find surface currents.

Geostrophic Shear

$$\frac{\partial}{\partial z}\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}(\rho f \mathrm{v}) = \overset{\mathrm{I}}{\cancel{f\mathrm{v}\frac{\partial \rho}{\partial z}}} + \overset{\mathrm{II}}{f\rho\frac{\partial \mathrm{v}}{\partial z}}$$

$$\frac{\partial}{\partial z}\frac{\partial p}{\partial y} = -\frac{\partial}{\partial z}(\rho f u) = \cancel{fu\frac{\partial \rho}{\partial z}} + f\rho\frac{\partial u}{\partial z}$$

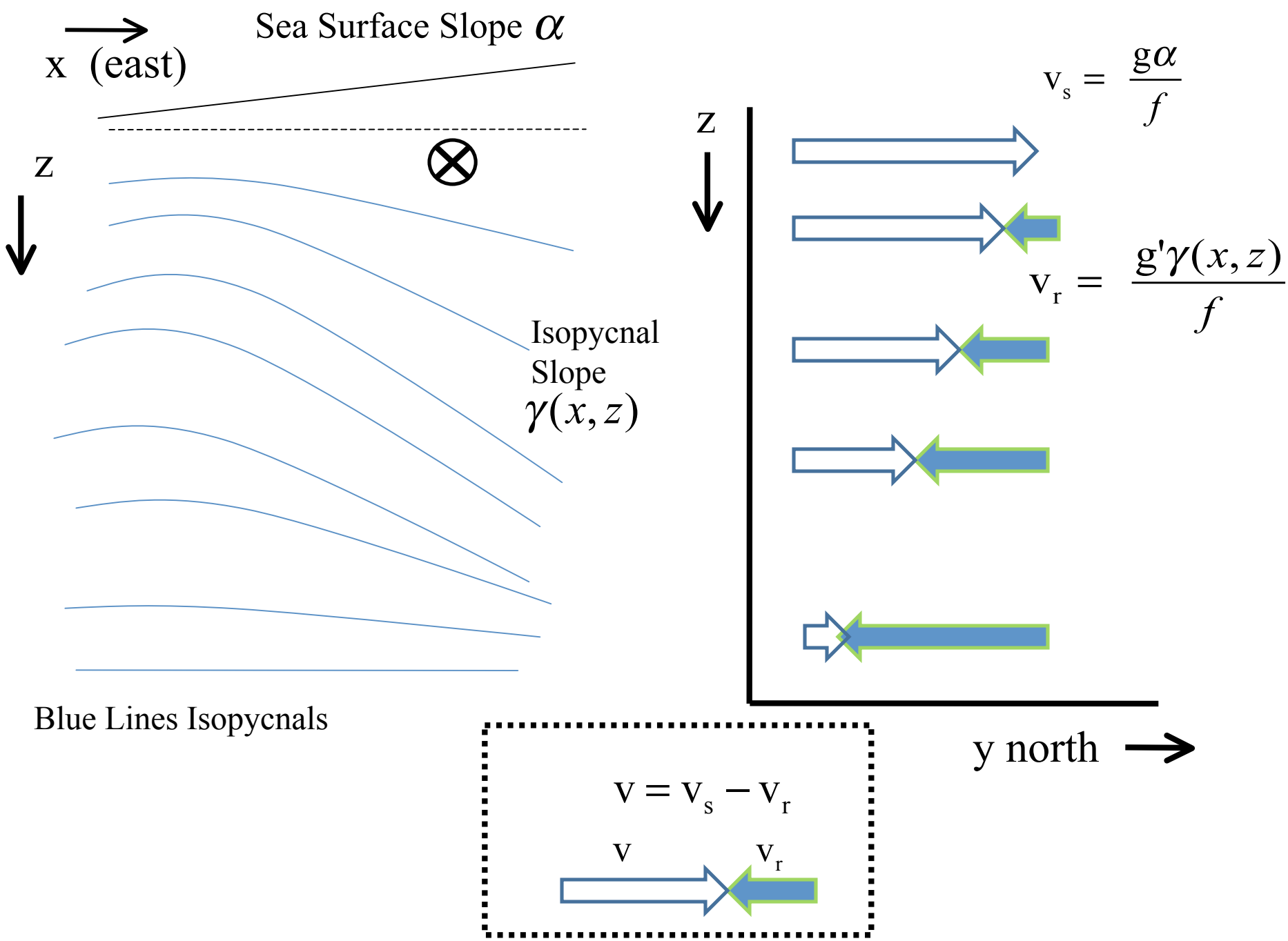
Homework: Use 'typical" large scale circulation values to show that term I is much smaller than term II

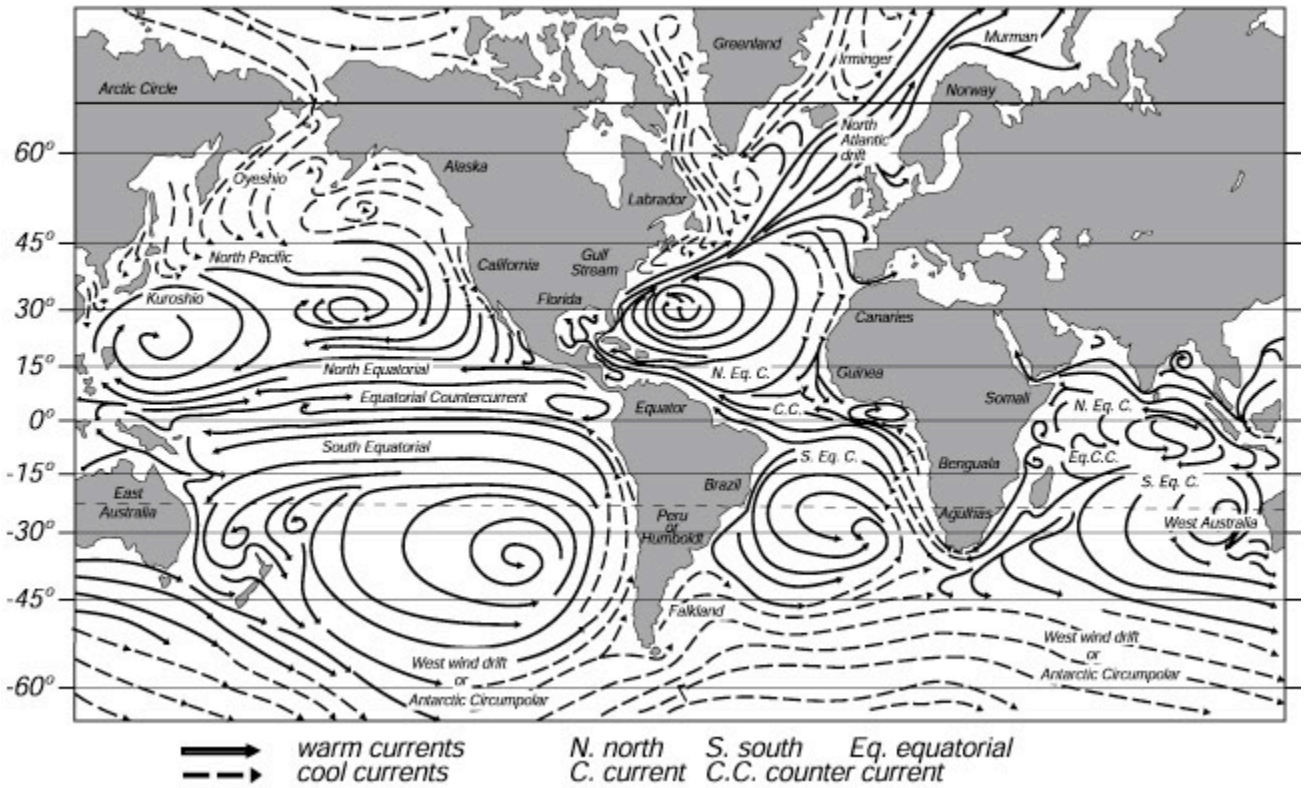
$$\frac{\partial}{\partial x}\left\{\frac{\partial p}{\partial z}\right\} = -g\frac{\partial \rho}{\partial x} \qquad \frac{\partial}{\partial y}\left\{\frac{\partial p}{\partial z}\right\} = -g\frac{\partial \rho}{\partial y}$$

"Thermal" Wind Equations

$$\Rightarrow \boxed{\frac{\partial \mathrm{v}}{\partial z} = \frac{g}{f\rho}\frac{\partial \rho}{\partial x} \qquad \frac{\partial u}{\partial z} = -\frac{g}{f\rho}\frac{\partial \rho}{\partial y}}$$

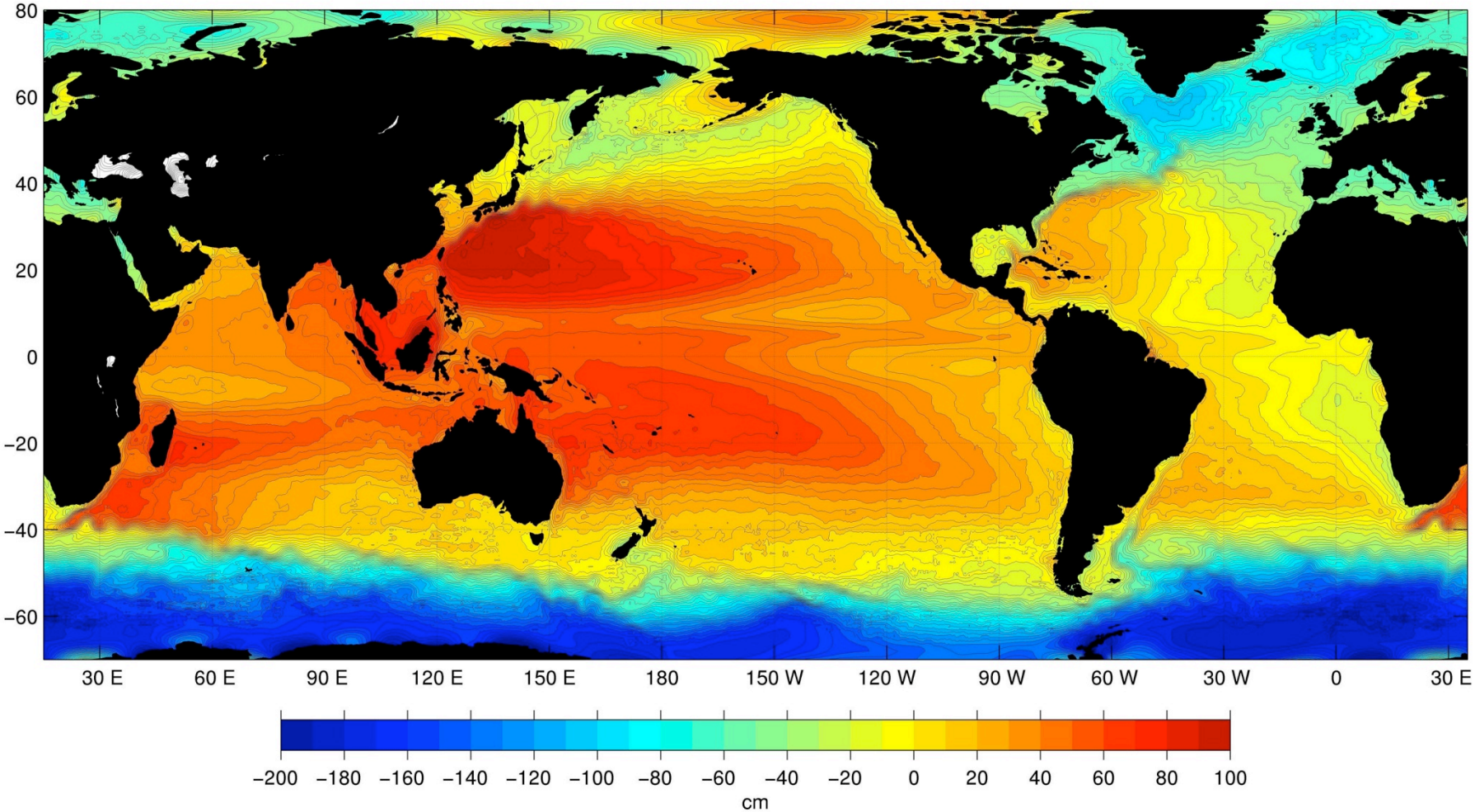
$$Note: \frac{\delta \mathrm{v}}{\delta z} \approx \frac{g}{f\rho}\frac{\delta \rho}{\delta x} \quad \Rightarrow \delta \mathrm{v} \approx \mathrm{v}_{\mathrm{r}} = \frac{1}{f} g \frac{\delta \rho}{\rho}\frac{\delta z}{\delta x} = \frac{g'\gamma}{f}$$


x (east)
Sea Surface Slope α
z
Isopycnal Slope $\gamma(x,z)$
Blue Lines Isopycnals
z
$v_s = \frac{g\alpha}{f}$
$v_r = \frac{g'\gamma(x,z)}{f}$
y north
$v = v_s - v_r$
v
v_r



<u>Older Traditional Approach to Estimating Surface Currents.</u>

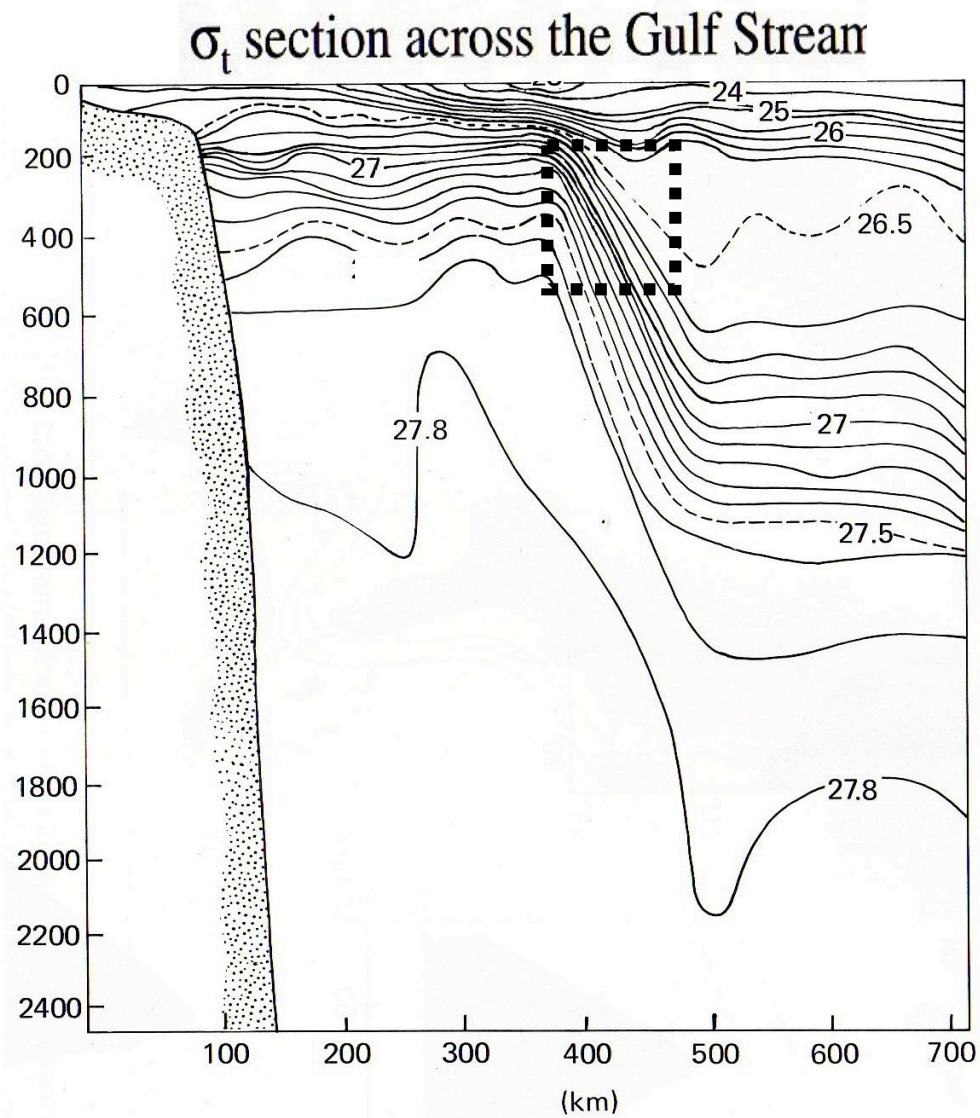
IPO figure 2.8. The time-averaged, surface circulation of the ocean deduced from nearly a century of oceanographic expeditions. From Tolmazin (1985). An assumption of a depth of no motion is made to obtain the surface velocity estimates from geostrophy.



Sea Surface Height (cm)

<u>Modern approach to estimating surface currents</u>

IPO Figure 10.5 Global, time-averaged topography of the ocean for the period 1992–2002 from a joint analysis of drifter, satellite altimeter, wind, and the GRACE Gravity Model-01 data by Nikolai Maximenko (IPRC) and Peter Niiler (SIO). Geostrophic currents at the ocean surface are parallel to the contours.



Homework: Using the concept of geostrophy and a level of no motion at z = 600m to estimate the surface current of the Gulf Stream. Use the dotted section for your calculation.

Conservation of Potential Vorticity and Deep Circulation

$$\frac{\partial p}{\partial x} = \rho f \mathrm{v} \qquad \frac{\partial}{\partial \mathrm{y}}\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial p}{\partial y} = -\rho f u \qquad \Rightarrow \rho\beta \mathrm{v} + \rho f\left(\frac{\partial \mathrm{v}}{\partial y} + \frac{\partial u}{\partial x}\right)$$

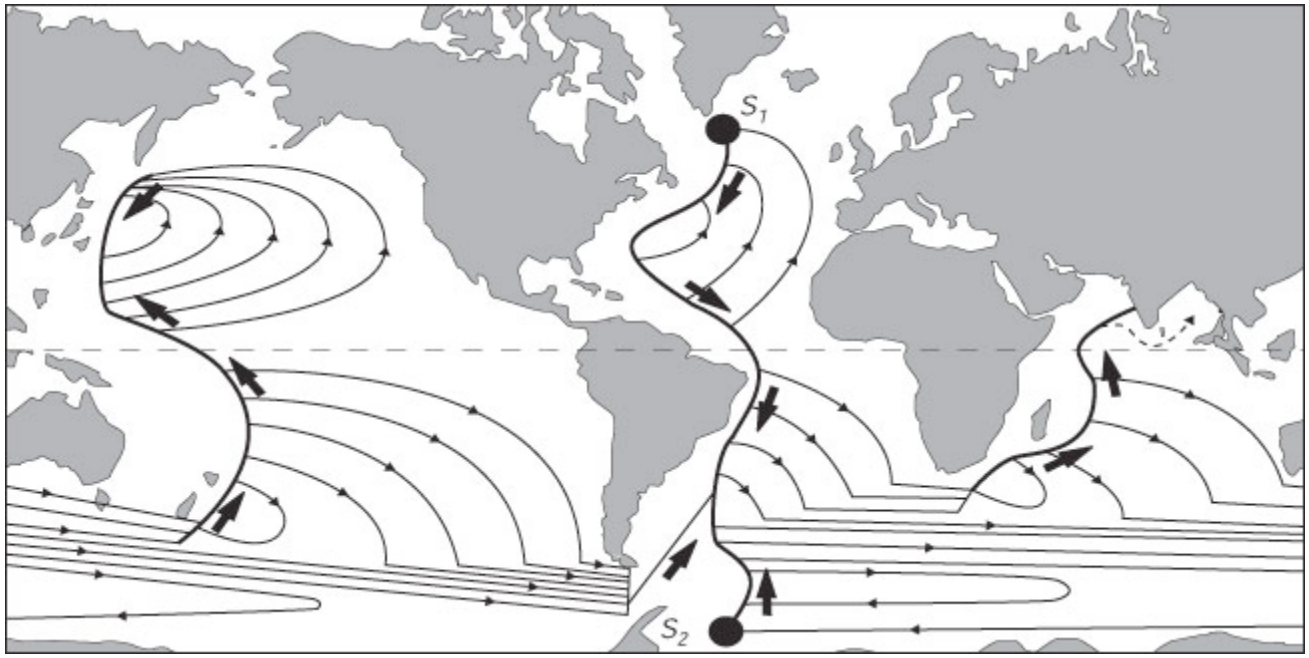
using $\frac{\partial \mathrm{u}}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} + \frac{\partial \mathrm{w}}{\partial z} = 0 \Rightarrow \boxed{\beta \mathrm{v} = \mathrm{f}\frac{\partial \mathrm{w}}{\partial z}}$

$$w = \frac{\beta}{f}\int_0^H \mathrm{v}\,dz = \frac{\beta}{f}V$$

or $\boxed{V = \frac{f w_H}{\beta}}$ H: height of some “layer” of water

V vertical intergal of northward velocity

Deep Circulation



Sketch of the deep circulation resulting from deep convection in the Atlantic (dark circles) and upwelling through the thermocline elsewhere. After Stommel (1958).

Why does the deep interior flow go equatorward in both hemispheres?

Answer $V = \frac{f}{\beta} w_H$. Explain.