Internal Waves

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Recap: Surface Gravity Waves

General dispersion with small amp (H/L <<1)

$$\omega^2 = gk\tanh(kh)$$

L/h < 1 → Short (deep water) waves

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$$

L/h > 1 → Long (shallow) waves

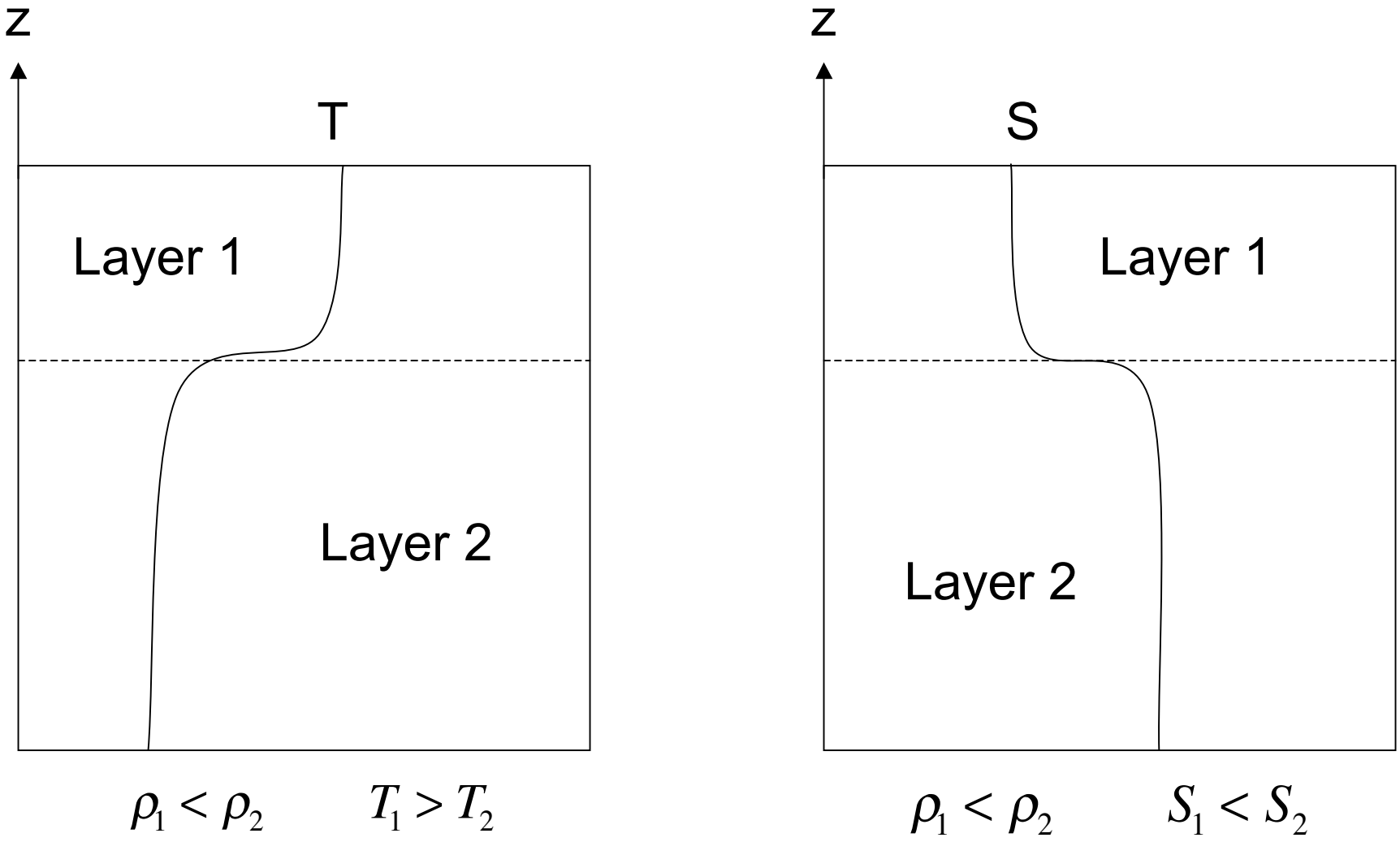
$$c = \frac{\omega}{k} = \sqrt{gh}$$

- -Power Spectrum
- -Sea state vs:
  - -Fetch
  - -Wind Speed
  - -Time
- -Refraction
- -Diffraction
- -Wave Power
- -Significant Wave Height
- -Average Period

Internal Waves

- We will first look at internal waves in a simple layered model with light water sitting on top of heavier water. This is a good approximation in many coastal areas. We will see the strong analogy with surface gravity waves.
- We will then consider the more complex theory of internal waves in a continuously stratified ocean. These waves do not necessarily move horizontally.
- Finally we will look at the influence of internal waves on ocean dynamics and ecology.

Layered Model

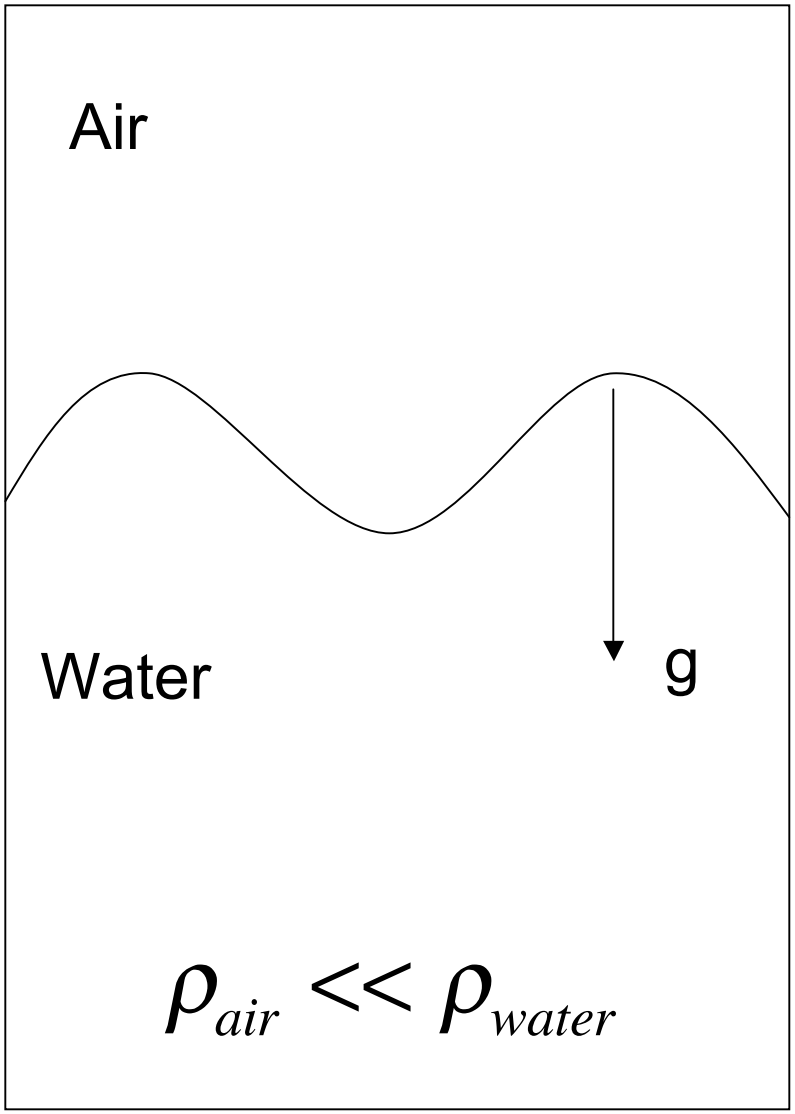


$\rho_1 < \rho_2$ $T_1 > T_2$

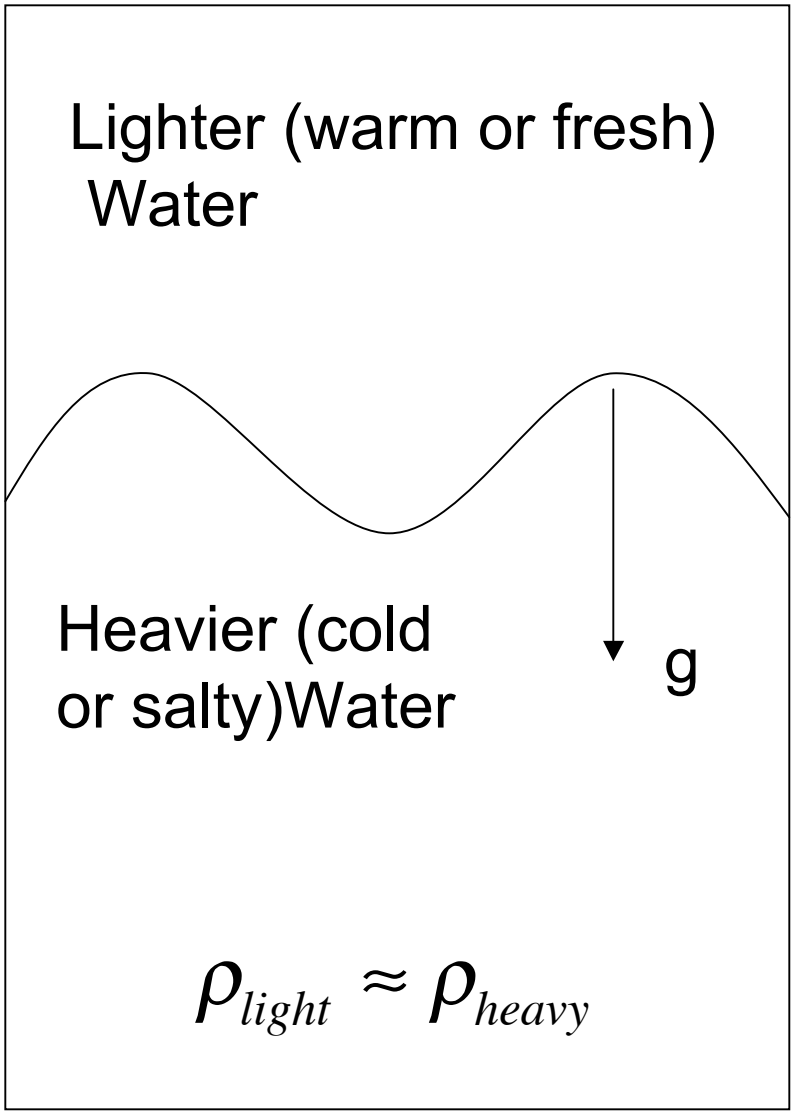
$\rho_1 < \rho_2$ $S_1 < S_2$

The ocean is not, in general, well-mixed and often can be represented by distinct layers of uniform density

Interfacial Waves: Comparison

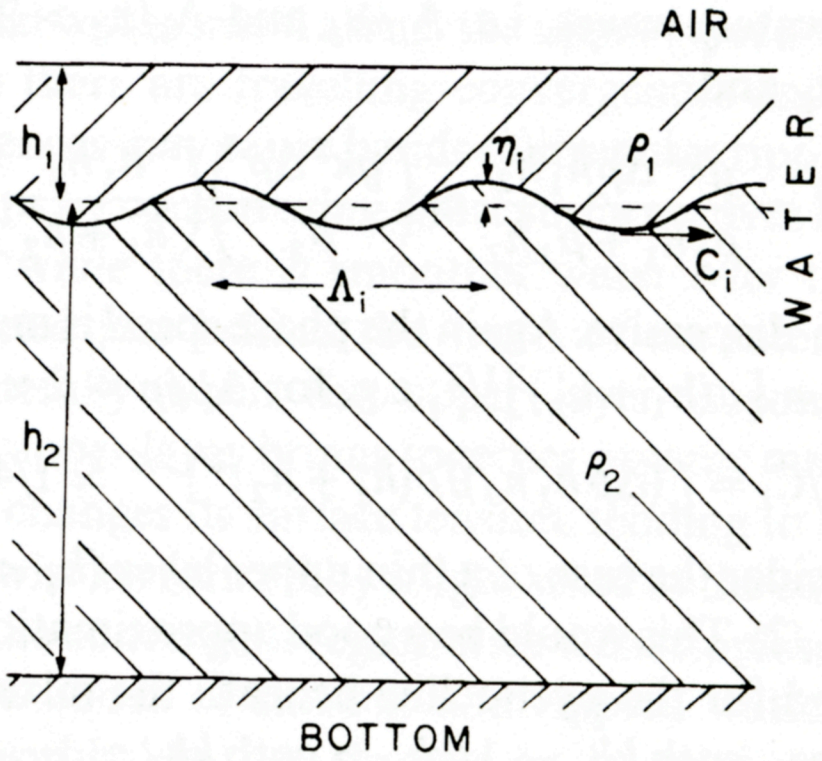


Surface Gravity Wave



Internal Wave

Internal Wave Speed



Using the approximation: $\dfrac{(\rho_2 - \rho_1)}{\rho_2} << 1$

We can calculate the speed of the internal interface C_i

Short Internal Waves

General Dispersion Relation

$$[\omega^2 - gk\tanh k(h_1+h_2)]\left[\omega^2 - \frac{(\rho_2-\rho_1)gk}{\rho_2\coth kh_2 + \rho_1\coth kh_1}\right] = 0$$

Limiting Case 1: Short (Deep Waves)

$$\lambda_i / h_1 < 1 \qquad \lambda_i / h_2 < 1$$

$$\omega_i^{\,2} = \left[\frac{\rho_2-\rho_1}{\rho_2+\rho_2}\right]gk; \qquad C_i^{\,2} = \frac{g\Delta\rho}{k(\rho_1+\rho_2)}$$

-Short Internal Waves are Dispersive

-Compared to Surface Gravity Waves they are very Slow!

$$for \quad \frac{\Delta\rho}{\rho} = 2e^{-3}: \qquad \frac{C_i}{C_s} = \sqrt{\frac{\Delta\rho}{\rho_1+\rho_2}} \cong 1/30$$

Long Waves

Waves are Not Dispersive (Analogous to Long Surf Waves)

$$\omega^2 \cong \left(\frac{gk^2\Delta\rho}{\rho_2}\right)\left(\frac{h_1 h_2}{h_1 + h_2}\right)$$

What about a thin upper layer and deep lower layer

$$C_i^{\,2} = \frac{g\Delta\rho h_1}{\rho_1}$$

Just like a surface gravity wave in depth h_1 with a speed reduction of

$$\sqrt{\frac{\Delta\rho}{\rho_1 + \rho_2}}$$

Example: S_1=0,S_2=30,T_1=T_2=10,h_1=5m: $\Rightarrow$**C_i = .8m/s**

Reduced Gravity

The key parameter in these relations is the reduced gravity g^*

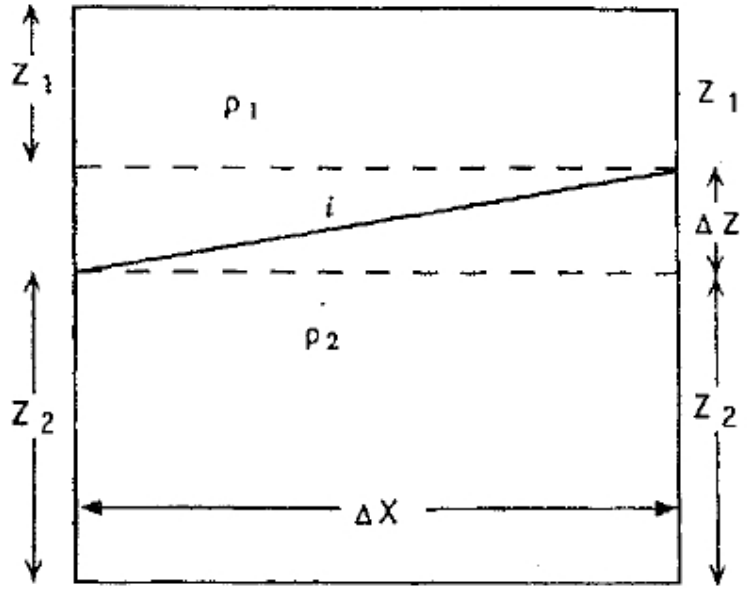
$$g^* = g\frac{(\rho_2 - \rho_1)}{\rho_2}$$

Celerity $$\frac{C_i}{C_s} = \sqrt{\frac{\Delta\rho}{\rho_1 + \rho_2}}$$ For both Long and Short

Energy $$E = \frac{1}{2}\rho g^* A^2$$

Physical Origin of Reduced Gravity: Pressure Gradients

Horizontal pressure gradient in our two-fluid system



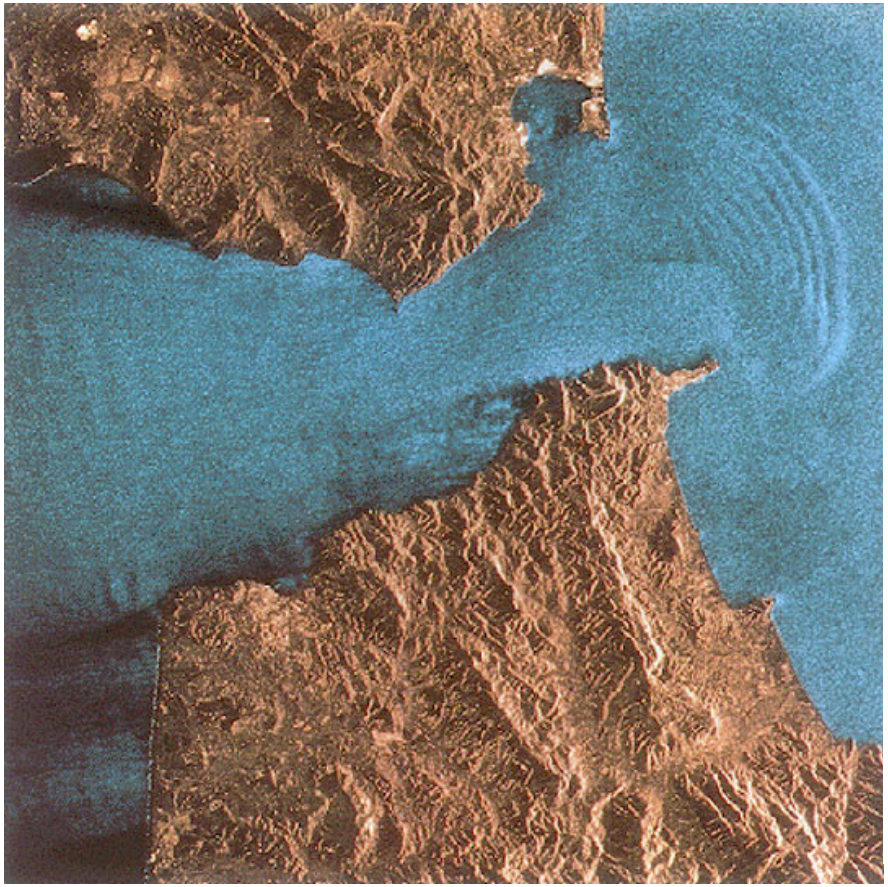
$$p_a = \rho_2 g z_2 + \rho_1 g(z_1 + \Delta z) \qquad p_b = \rho_2 g(z_2 + \Delta z) + \rho_1 g z_1$$

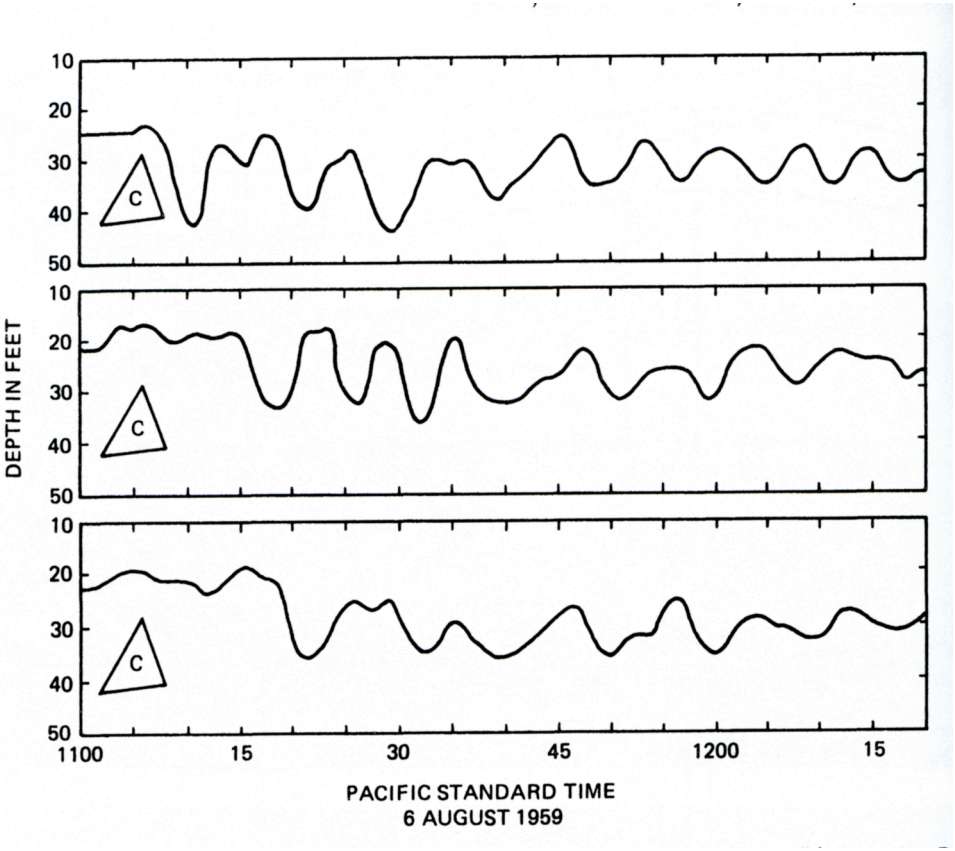
$$\frac{1}{\rho}\frac{dp}{dx}_{bottom} = \frac{(\rho_2 - \rho_1)}{\rho_2} g \frac{\Delta z}{\Delta x} = g^* \frac{\Delta z}{\Delta x}$$

Recall: Surface Pressure Gradient: $\frac{1}{\rho}\frac{dp}{dx}_{bottom} = g\frac{\Delta z}{\Delta x}$

Our driving (restoring) force is significantly reduced!

Internal Wave Observations





Strait of Gibraltar: $\lambda = 750 - 1000m$

Surface roughness changed by wave-current interaction. Detected by SAR Satellite Sensor

Off San Diego: $T_p = 300s$

Signal is depth of 64F isotherm measured by a moored station with vertical array of CTDs

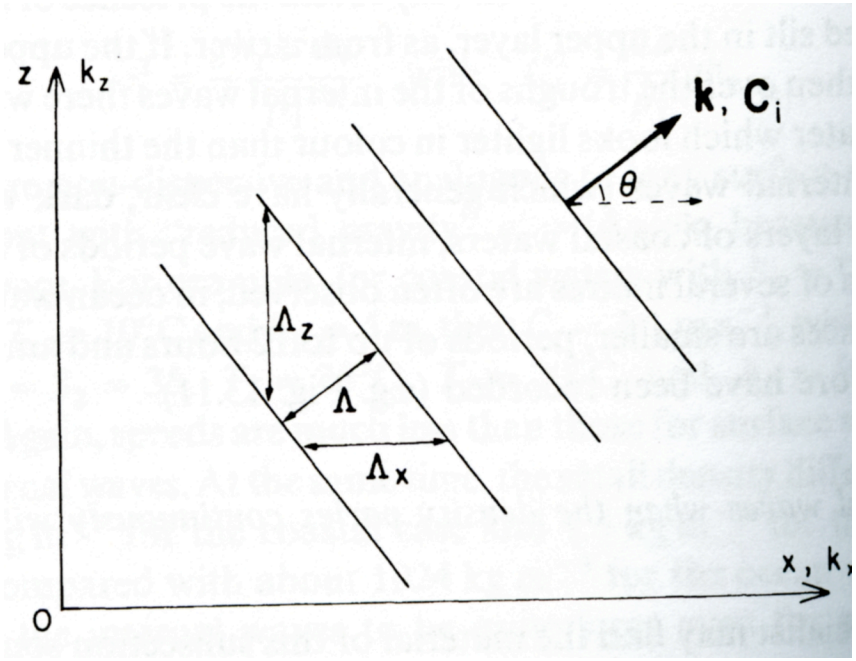
Summary: Two-Layer Internal Waves

- Two Layer model allows Interface Waves (of which surface gravity waves are a subset)
- Long waves are non-dispersive while short waves are dispersive (same as surface gravity)
- Reduced gravity factor diminishes speed compared to surface gravity waves.
- Reduced gravity effect derives from decreased horizontal pressure gradient between perturbed layers of similar density compared to the huge density difference between air and water.
- Waves move horizontally (lat/lon)
- Reduced gravity allows for large amplitudes compared to surface gravity waves
- Internal Waves can influence surface signature: detectable by satellites

Internal Waves in a Continuously Stratified Fluid

- Previous discussion was for a two-layered system which is quite realistic for coastal systems
- The real ocean however, is not a two-layer fluid: Density increases continuously with depth
- Internal waves in a continuous medium can move horizontally and *vertically!*.

Internal Wave Celerity



N is the upper limit on the wave frequency!

Brunt-Väisälä Frequency (N)

$$N^2 \cong \left(-\frac{g}{\rho}\right)\left(\frac{\partial \rho}{\partial z}\right)$$

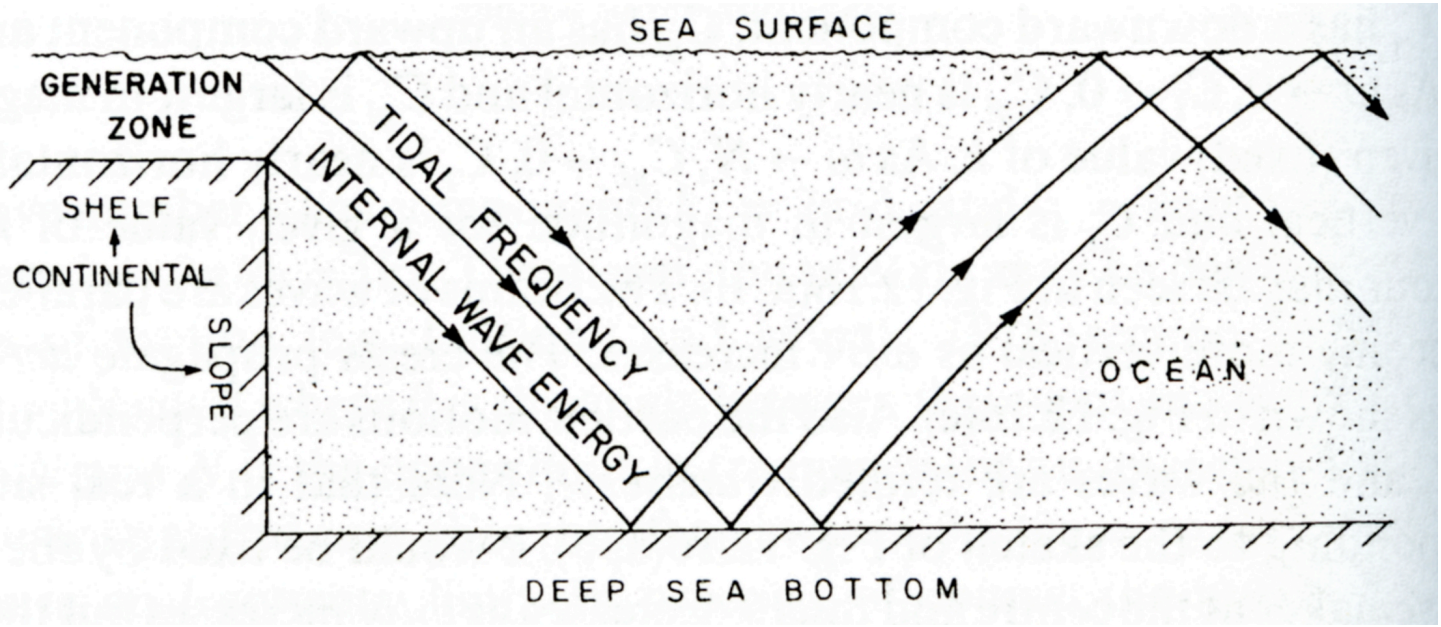
Frequency $\omega = N\cos\Theta$

Celerity $C_i = \dfrac{N\cos\Theta}{k}$

<u>Depends on:</u>

- Properties of the medium (N)
- Wavenumber (is dispersive)
- Direction!

Refraction/Reflection



- N generally decreases with depth
- At the surface and bottom N=∞ and we have total reflection
- Analogously to the surface wave obliquely approaching a beach, the wave will be steered when propagating obliquely across regions of changing N.

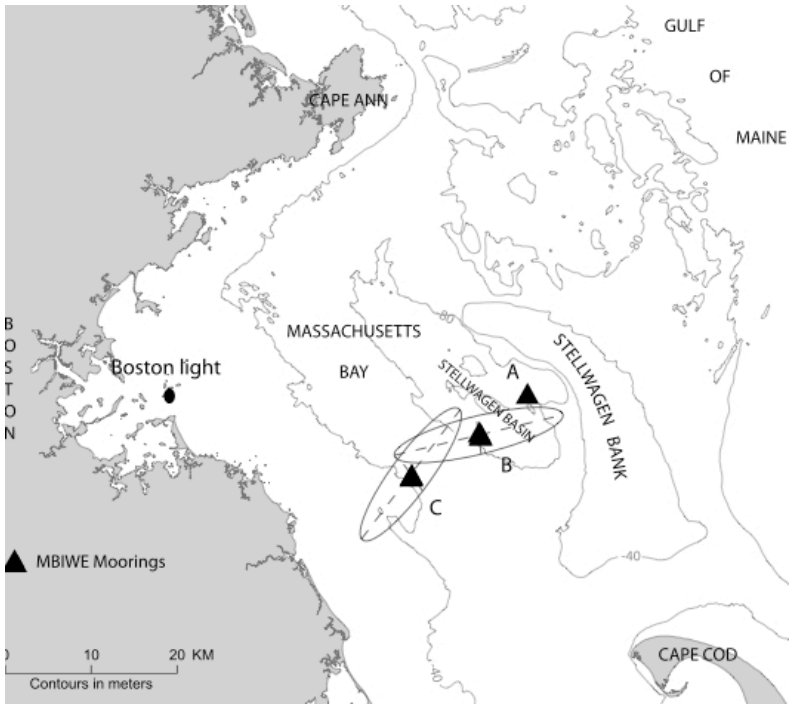
Internal Wave Aspects

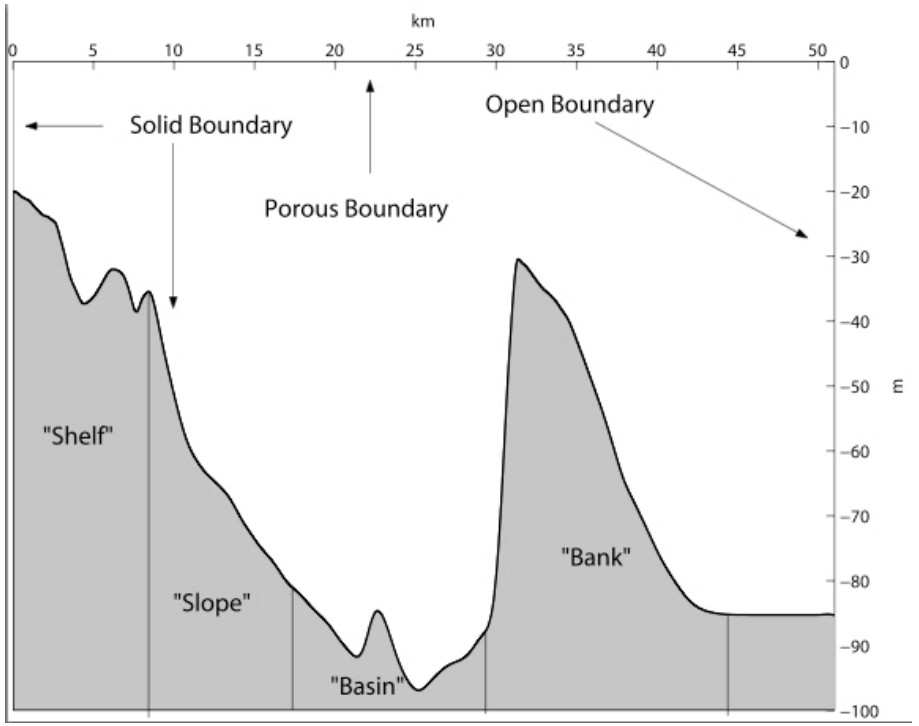
1. Generation: Some Understanding
2. Propagation: Theoretical Foundation
3. Dissipation: Poor Understanding

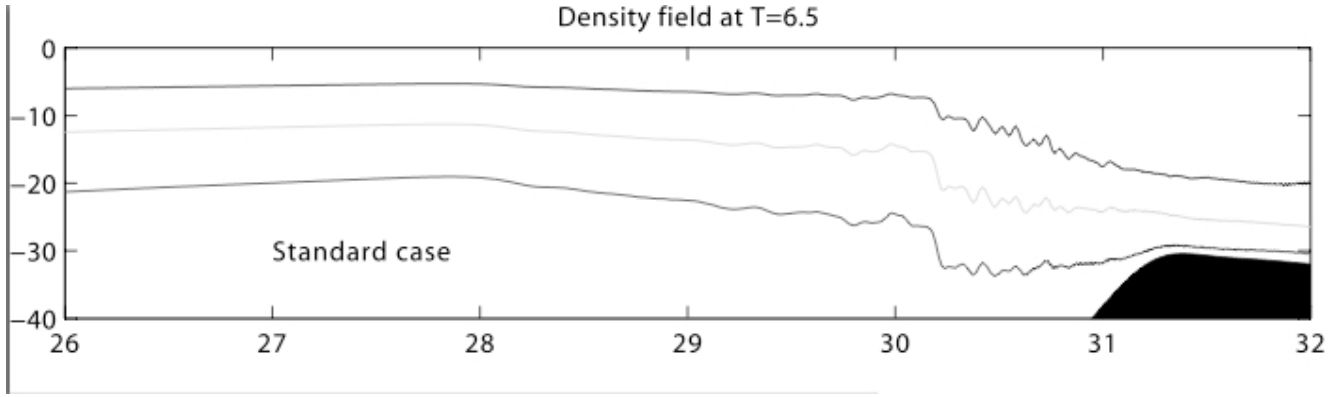
<u>Generation Mechanisms</u>

- Stratified flow forced over abrupt bottom topology (can be tidally forced “internal tides”)
- Atmospheric pressure variations

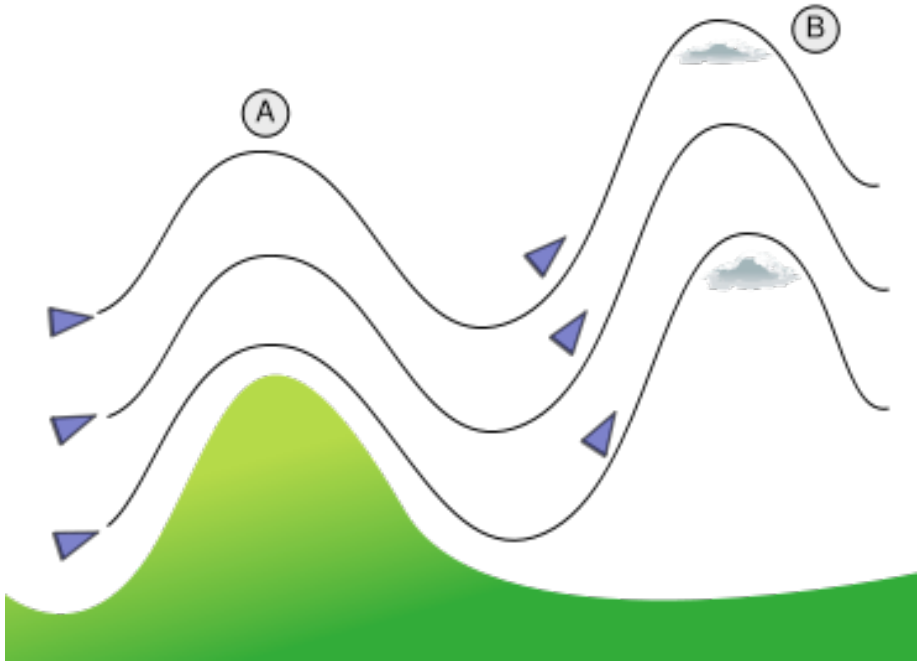
Stellwagen Bank







Internal Waves in the Atmosphere



What role to internal waves play in the ocean?

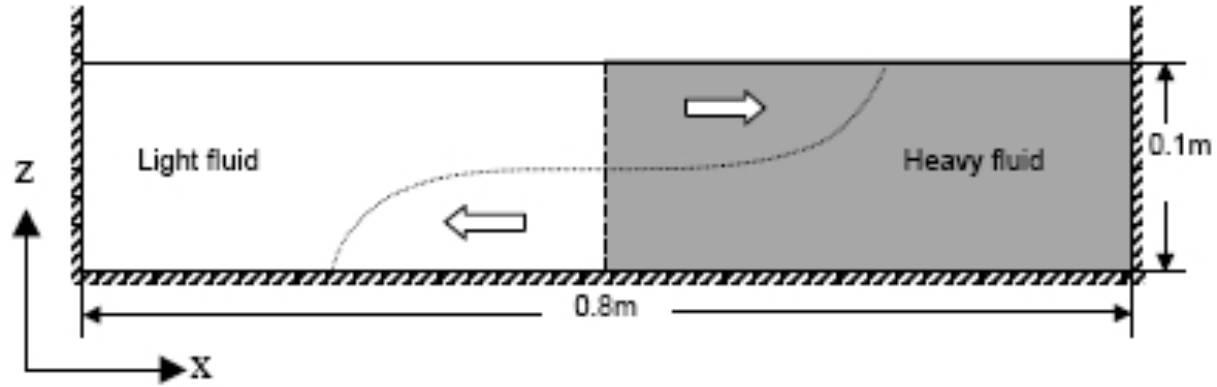
Mixing: Enhanced Vertical Exchange

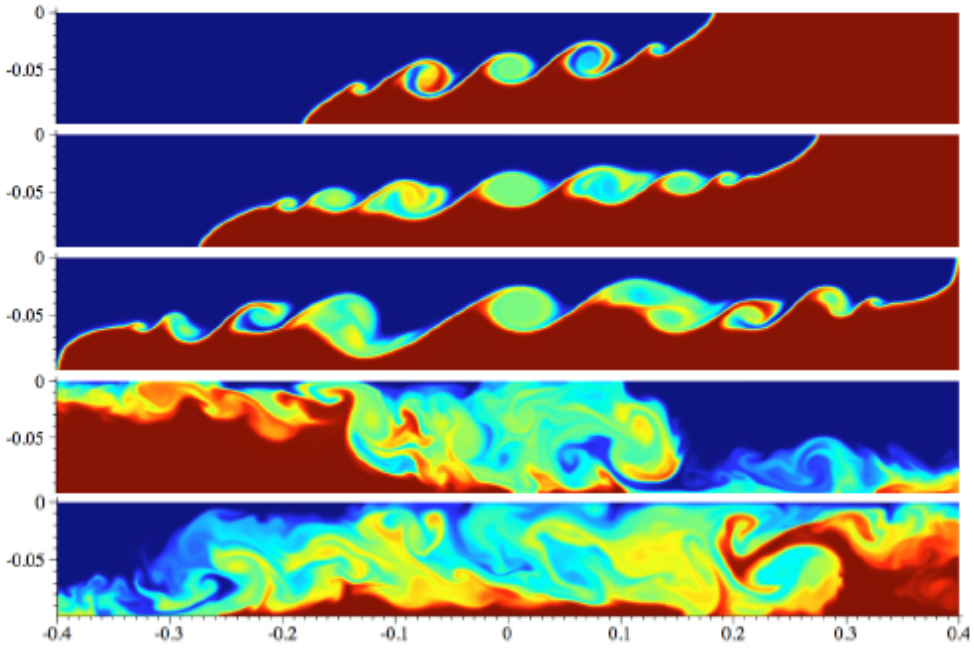
- Overturning and displacement of pycnoclines leads to enhanced mixing in the vertical.
- Overturned eddies can entrain deep water into the surface layer.
- Although important, the process is very complex and difficult to include in numerical models of the ocean.

Hydraulic Control:

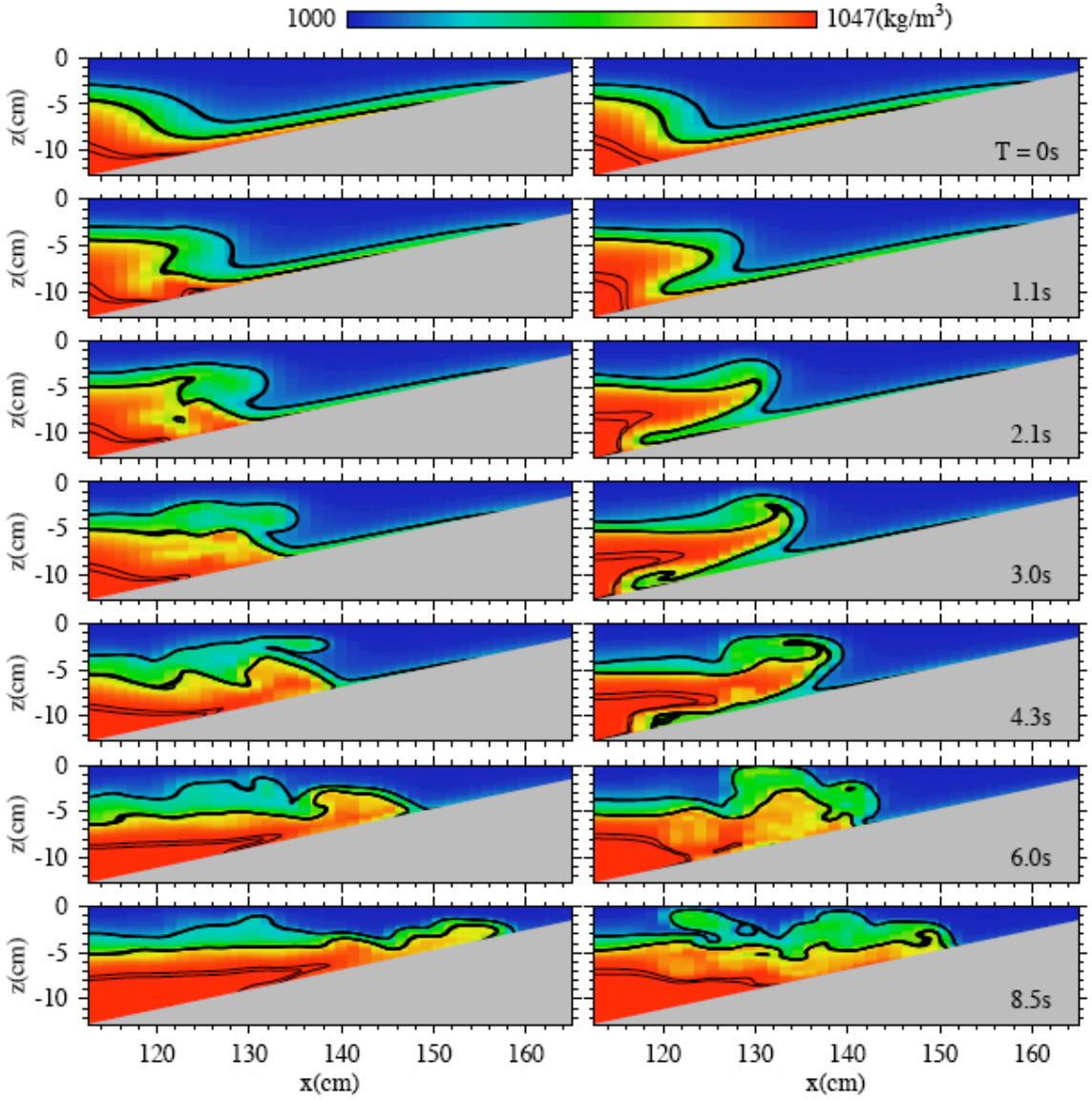
- If the current is stronger than the wavespeed, internal waves cannot propagate upstream.
- This often happens when water is constricted by a narrowing coastline or a sill.
- This can influence the exchange of water between two bodies and is known as ‘hydraulic control’

Mixing (Extreme Example)





Interaction with Coast



How do internal waves influence ecology?

- Shoaling internal waves can aggregate nutrients. In the South China Sea, Pilot Whales are known to track and follow internal waves to forage on prey aggregations.
- Breaking internal waves can be a significant source of nutrients from the deep water to the eutrophic zone.
- Internal wave breaking near the bottom can produce scouring events representing a significant natural disturbance event to the benthos (e.g. Stellwagen Bank)

Questions

- In water of depth 200m, a 7m layer of fresh (20 PSU) estuarine water of temperature 20C overlies a layer of saltier (30 PSU) ocean water, of temperature 25C. What is the celerity of long internal waves propagating along the interface. How does this compare to the speed of a long surface gravity wave in the same medium.

Question 2

- In a region where density varies linearly with depth, internal waves are observed propagating horizontally with a period (T) of 300s. What is the density gradient (approximately) in the water column.