# Waves: Effects of Rotation

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# Recap: Surface Gravity Waves

General dispersion with small amp (H/L <<1)

$$\omega^2 = gk\tanh(kh)$$

L/h < 1 → Short (deep water) waves

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$$

L/h > 1 → Long (shallow) waves

$$c = \frac{\omega}{k} = \sqrt{gh}$$

- -Power Spectrum
- -Sea state vs:
  - -Fetch
  - -Wind Speed
  - -Time

- -Refraction
- -Diffraction
- -Wave Power
- -Significant Wave Height
- -Average Period

# Recap: Internal Gravity Waves in a Layered Ocean

## Comparison with SGWs

- Surface Gravity Waves are a type of Internal Gravity Wave between two vastly different fluid densities (1000/1)
- Internal interface waves propagate between two fluids if nearly equal density
- Both short wave and long wave limits exist and, not surprisingly, the dispersion and celerity formulations are similar to surface gravity waves

## Key Parameter: Reduced Gravity

- Pressure Gradient for internal interface slope: g' deta/dz
- For a given pressure gradient, internal interface slope much larger than surface slopes: This allows internal waves to be much larger in amplitude in general
- Propagation speeds are considerably smaller, roughly 1/30th that of surface gravity waves for a typical two layer system.
- Reduced propagation speed can lead to hydraulic control: Local currents greater than wavespeed, wave cannot propagate upstream!

# Recap: Internal Gravity Waves in Continuously Stratified Ocean

## Key Parameter: Buoyancy Frequency (N):

- Waves can move vertically as well as horizontally
- Celerity depends on Buoyancy Frequency (N) as well as propagation angle relative to horizontal
- Dependency on Buoyancy Frequency (a measure of density gradient) means waves will be subject to refraction when propagating with some vertical component, they will tend to bend and curve in space.
- At the f-s and bottom boundaries, N is effectively infinity and wave will reflect off these boundaries.
- N is the upper limit on frequency of the internal wave

## Internal Waves and Mixing

- Internal Waves can enhance diapycnal mixing through breaking and entrainment
- Shoaling of internal waves formed by interaction of tides and steep topography in stratified water (e.g. Stellwagen Bank) can lead to enhanced scouring of bottom, that is strong natural disturbances at tidal frequencies.
- In general, propagation well understood, formation fairly mature, but dissipation mechanisms and influence on local circulation up to global ocean still a subject of intense study.

# Large-Scale Motions

- We will see large-scale waves whose period is near the inertial period exist
- These waves come in various types influence the presence of boundary, and different types of restoring forces
- Ultimately these waves are responsible for the adjustment of the ocean in response to changes in atmospheric forcing

# When is Rotation important

Acceleration:

U varies as $\sin(2\pi t / T)$

$$\frac{\partial u}{\partial t} \simeq \frac{2\pi}{T} u$$

Coriolis:

$$fu$$

When wave period approaches pendulum half-day..

$$T \simeq 2\pi / f$$

*Coriolis and Acceleration are of same order!!*

# Baseline Solution:

Constant Density/Depth

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \qquad \frac{\partial v}{\partial t} + fv = -g\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$u = [(g/h)(1 - s^2)]^{1/2}\,\eta\cos(kx - \omega t)$$

$$s = fT/2\pi \qquad \omega^2 = (ghk^2 + f^2)$$

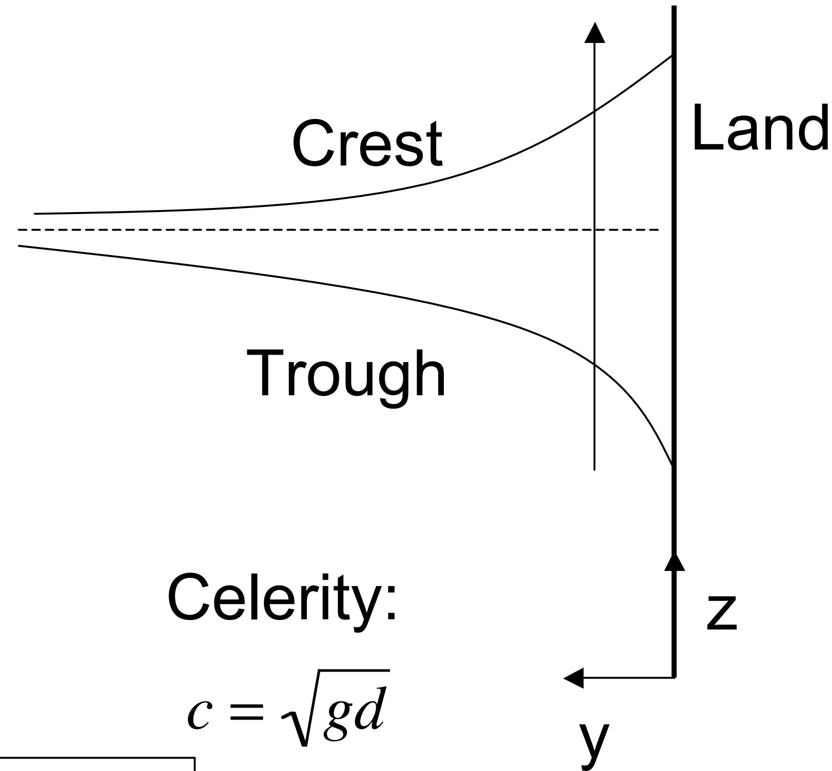
$$C = [gh/(1 - s^2)]$$

# Rotation Effects on Constant Depth/rho Wave Solution

- Waves are known as Poincare waves
- Waves are dispersive when rotation effects (s) large (~1)
- Rotation introduces transverse motion
- Celerity increased over long surface wave with no rotation
- Group velocity < Phase velocity when s large
- Sometimes known as Sverdrup Waves

# Lateral Boundary

## Inviscid/Linear N-S Equation + rho/h constant




Cross-shore Momentum Equation:
Geostrophic Balance

$$g\frac{\partial \eta}{\partial y} = -fU$$

The general solution
for the amplitude is:

$$\eta = \frac{H}{2} e^{-fy/C} \cos(kx - \omega t)$$

Celerity:

$$c = \sqrt{gd}$$

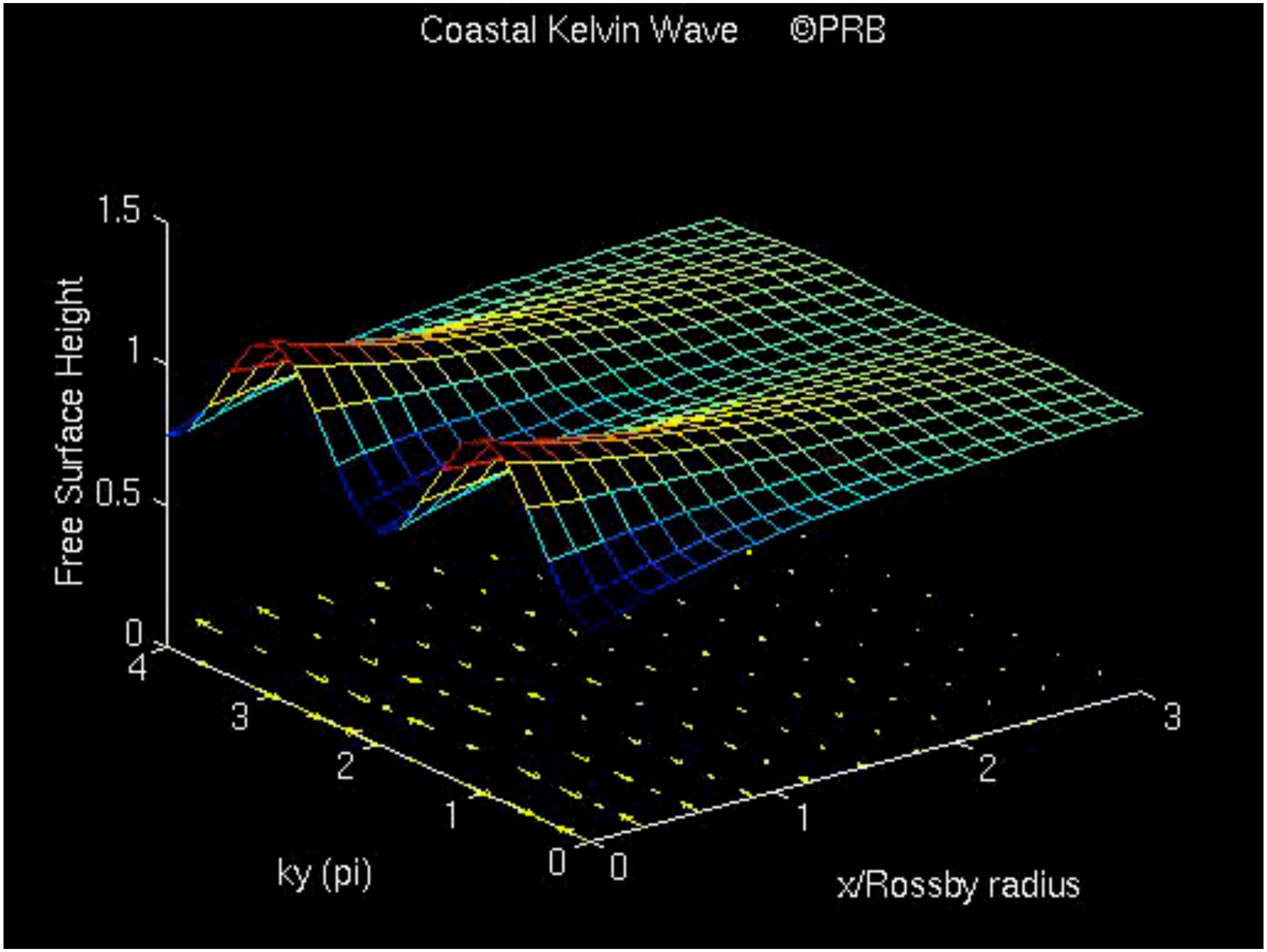
Velocity:

Decay Scale $C/|f|$

$$U = \frac{H}{2}\frac{C}{d} e^{-yf/C} \cos(kx - \omega t); V = 0$$

Solution first derived by Lord Kelvin: Is is a Kelvin Wave!

# Structure




Credit?

# Kelvin Waves

- Geostrophic balance between Coriolis from alongshore motion and cross shore pressure gradient
- Very Important:  Waves travel in Northern Hemisphere with Coast to the right.
- Kelvin waves are non-dispersive, energy moves with phase velocity
- Amplitude of the wave decreases away from shore exponentially with e-folding scale of $r = c/f$ where $c$ is the phase speed and $f$ is Coriolis parameter.  (This distance is the relevant Rossby Radius)

# Internal Kelvin Waves

- Pressure gradient from reduced gravity
- Phase speed - (g*h)^(1/2) based on reduced gravity
- Scale of amplitude decay (Rossby Def Radius) smaller

$$\lambda \simeq C / |f| = (g^* h_1)^{1/2} / |f|$$

Example Decay Scales - Barotropic Kelvin Wave:

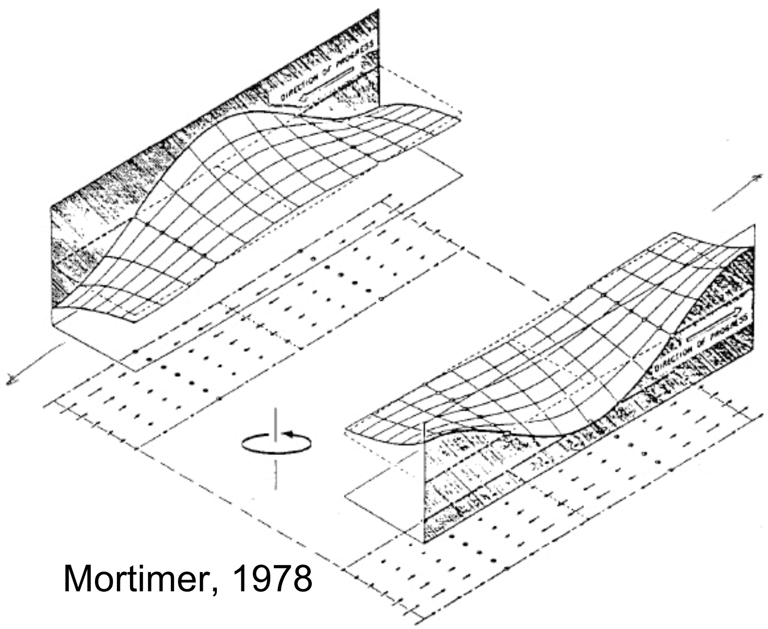
Deep Water: h = 5km , C = 220m/s, f=1e-4, C/|f| ~ 2000 km

Shallow Water: h = 100m , C = 30m/s, f=1e-4, C/|f| ~ 300 km

Example Decay Scales - Internal Kelvin Wave:

h = 150m , g*=.02, C = 3m/s, f=1e-4, C/|f| ~ 30 km

# Kelvin Waves: Tides



Mortimer, 1978

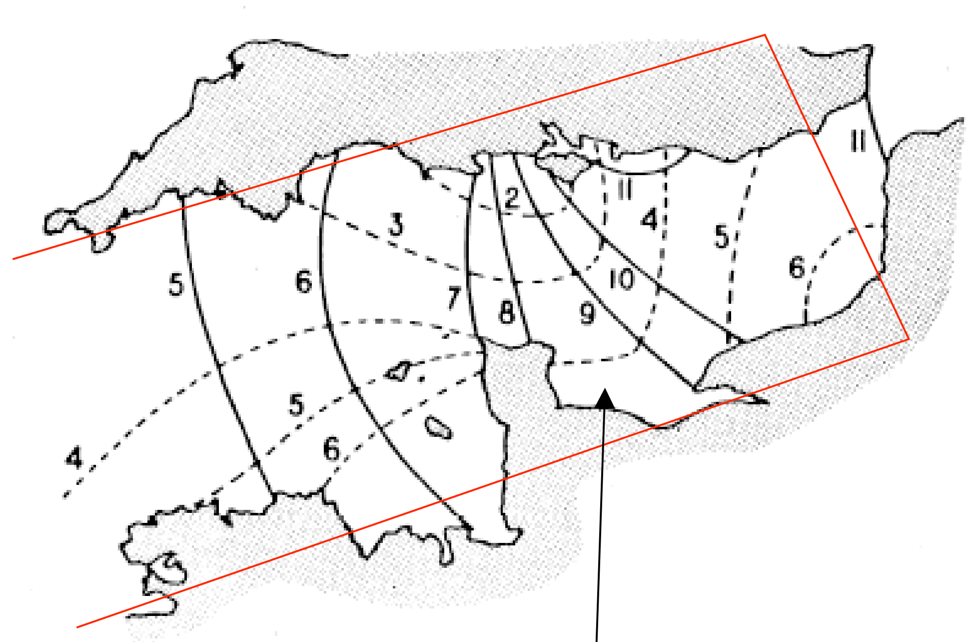
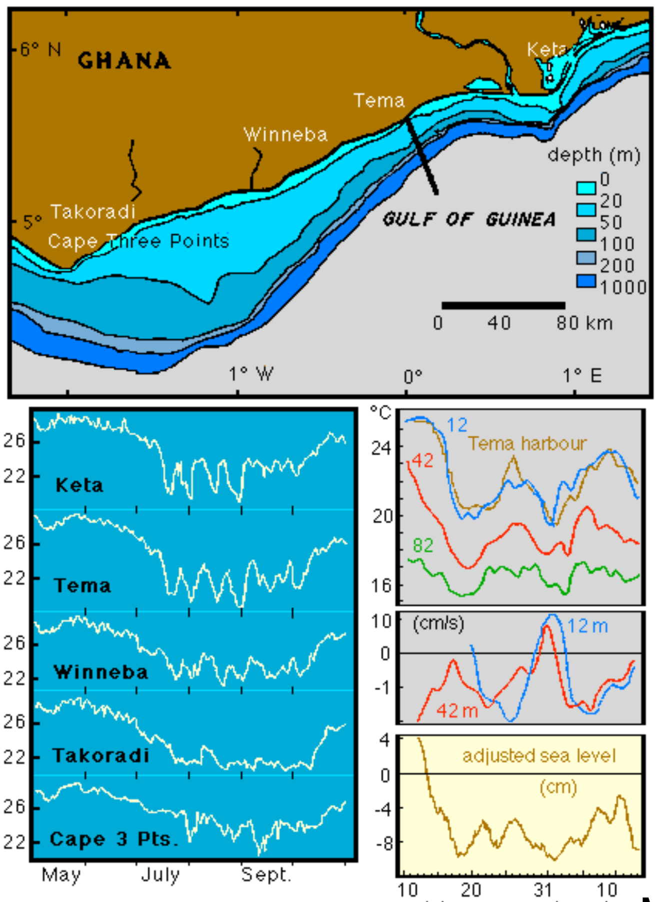


**Fig. 10.5.** Co-tidal lines (solid) with time in lunar hours, and co-range lines (dotted with
the English Channel. [From Proudman (1953, p. 262); after Doodson and Corkan (1931).]

Amplitude higher on
French “right” coast.

# Coastal Kelvin Waves:

Require a Coast to “lean against”, although steep topography (a shelf) may also support wave propagation.




adapted from Houghton and Beer (1976)

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Example Perturbation: Storm Passage

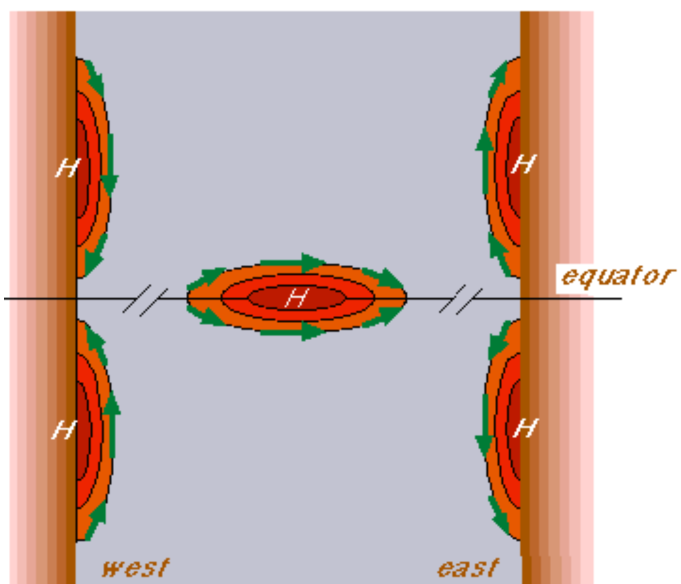
-Waves form and follow coast

- Notable on Cali Coast

- Visible in Temperature record and low-passed filtered currents here for Ghana coast

# Equatorial Kelvin Waves

- The zero crossing of f serves a lateral boundary
- Geostrophic Balanced across the equator
- A pair of Kelvin waves effectively leaning against each other is dynamically supported
- Waves only can propagate from west to east
- Energy follows the phase speed of the wave

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# Rossby (Planetary) Waves

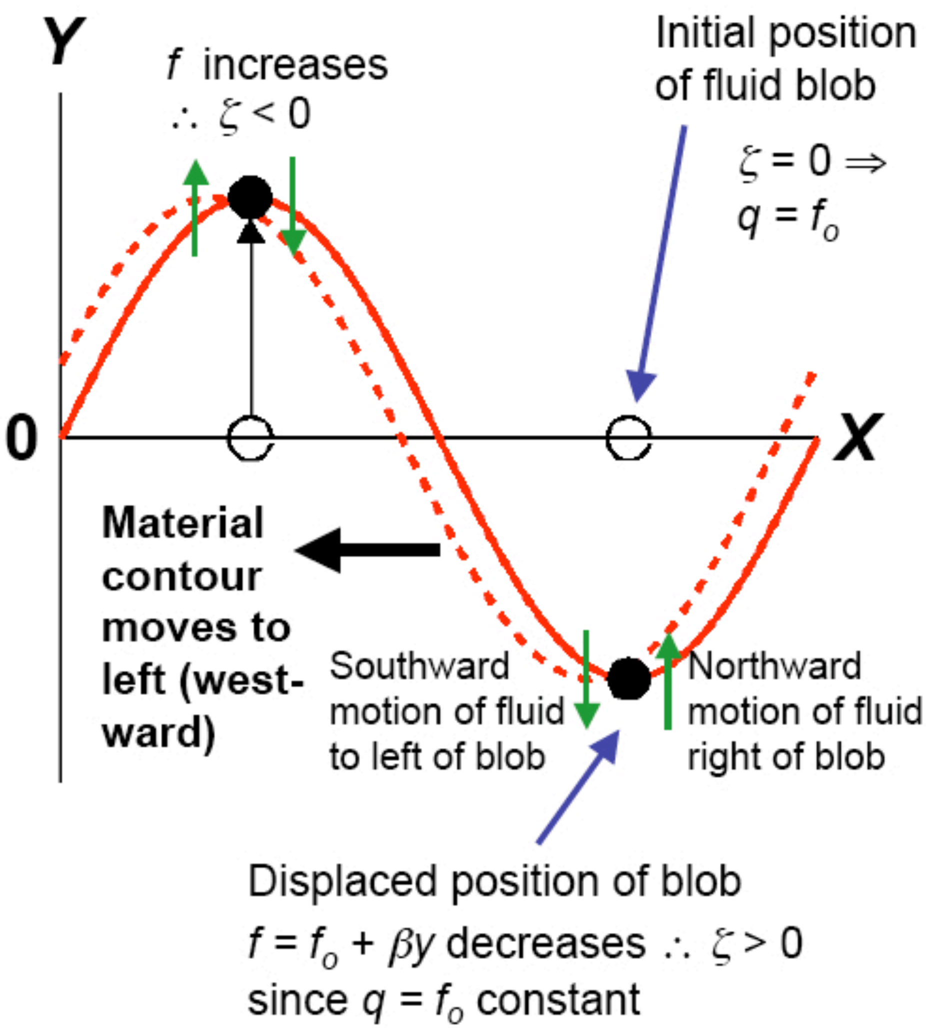
- Initial perturbation in meridional direction leads to westward propagation of Rossby Waves Waves are
- Most obvious: Fronts on a weather map.
- Key: Restoring force is NOT gravity, but the change in f
- Shape of planet, not just rotation, plays a role!
- In the ocean, are hard to detect, have huge horizontal scales of O(100km), but small vertical (O(cm)).
- Change in f with latitude fastest near the equator.

$$f = 2\Omega \sin\phi$$

$$f = f_0 + \beta y$$

Linearize: around a given latitude: *Beta* plane Approximation

# Rossby (Planetary) Waves




*McLandress*

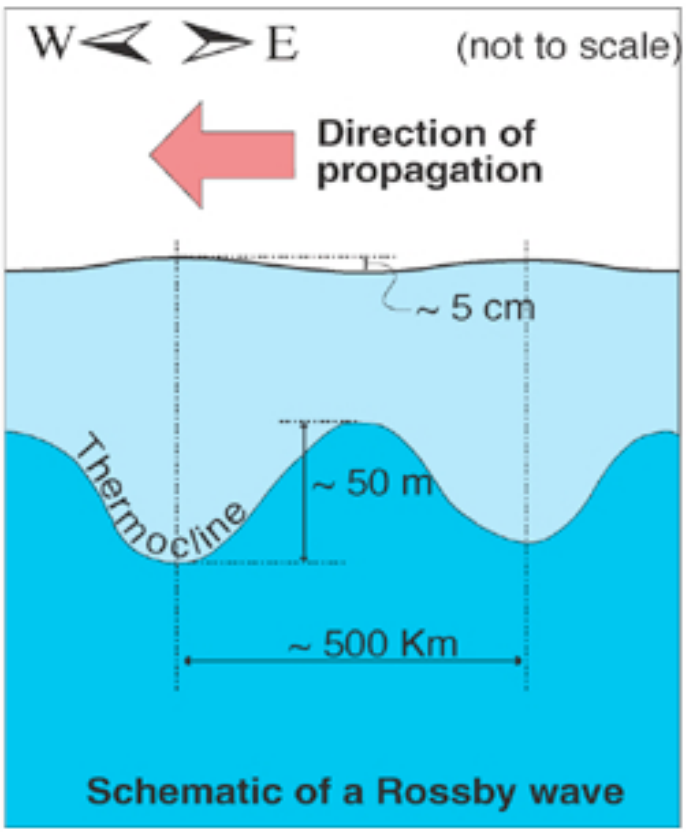
Key is conservation of potential vorticity $PV = \dfrac{\xi + f}{D}$

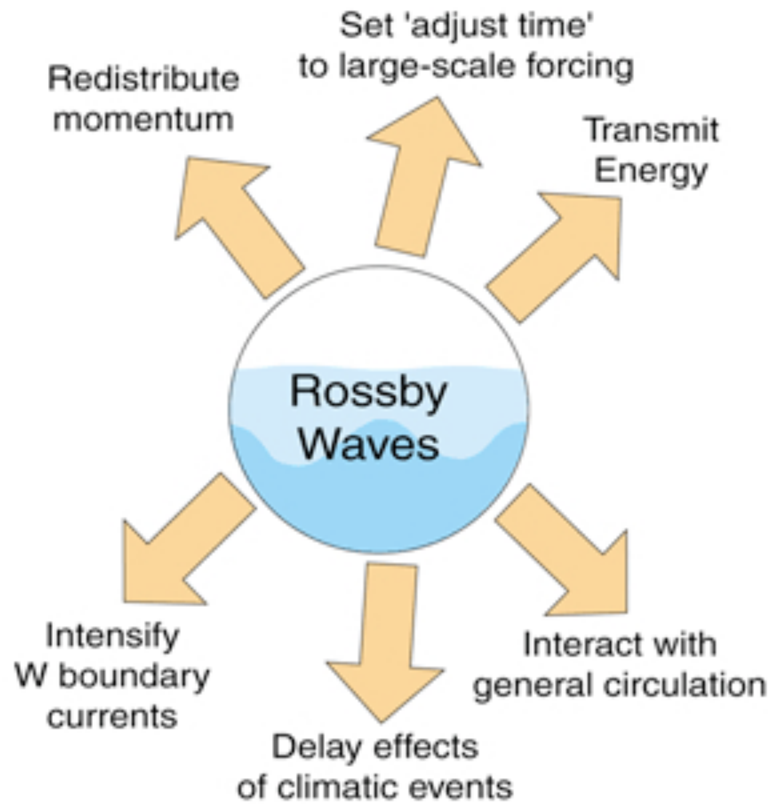
Consider some initial perturbation around a line of constant latitude.

Restoring force is generated supported westward wave propagation.

Note that changes in D can also support waves (Topographic Planetary Waves)

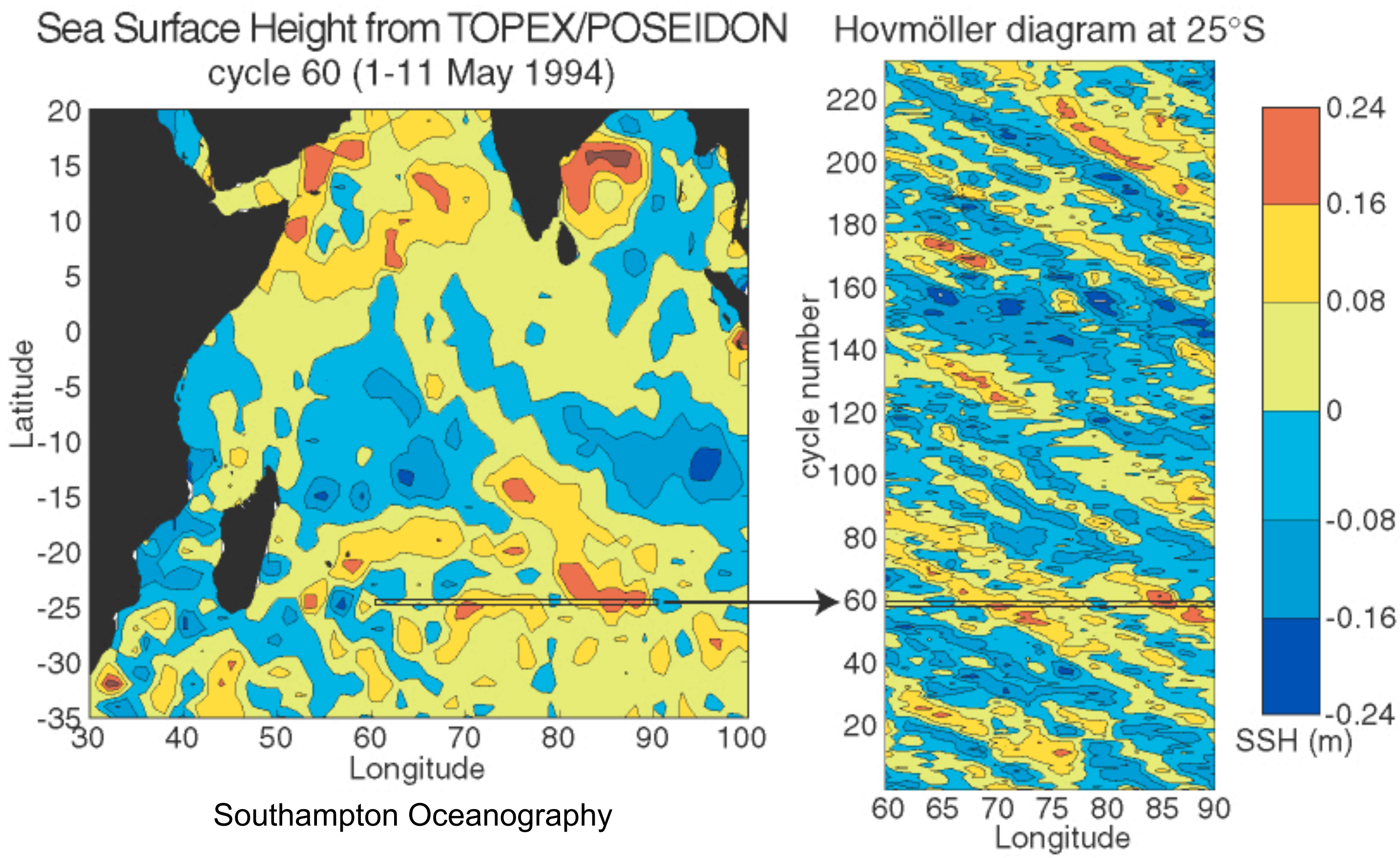
# Rossby Waves: Large Scale Adjustment



# Observing Rossby Waves

Surface Signature: Very small (cm) against a very noisy background. However, as they primarily move zonally, can use a trick.

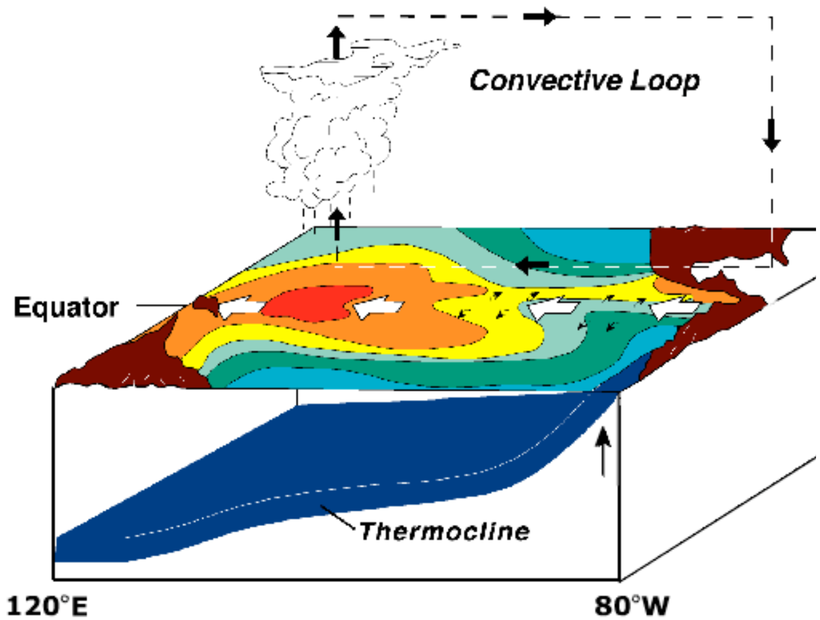



Southampton Oceanography

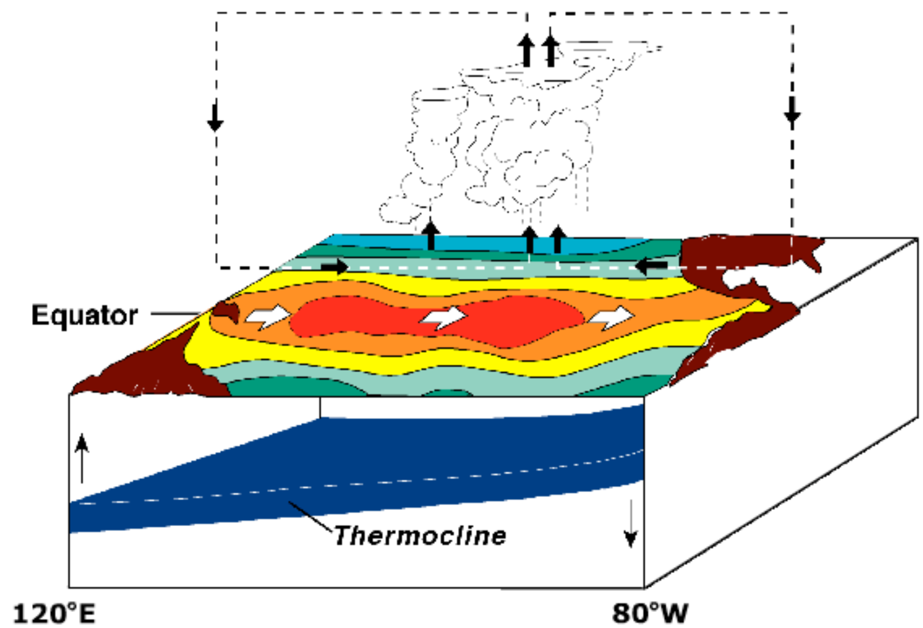
Each satellite pass (cycle), compute SSH anomaly and plot vs. Longitude: See the streaks of crests (24cm) and troughs (24cm) move westward with time. Simply Amazing.

# Large-Scale Ocean Adjustments: El Nino

Rossby and Kelvin Waves set the adjustment time of the ocean to changes in atmospheric forcing.

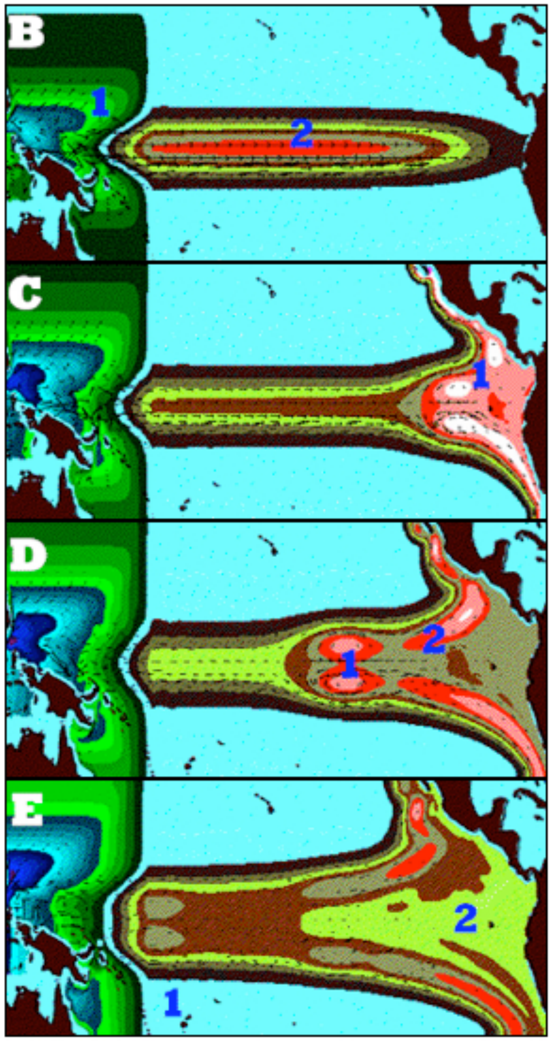



Normal




El Nino

# Equatorial Adjustments: El Nino




- Easterly Winds Relax

Adjustment: Kelvin Wave/Rossby

1. Kelvin Moves moves to the East at 3 m/s (across Pacific in 70 days)
2. Reflects off South America as westward moving Rossby Wave (at 1 m/s, 210 days across Pacific)
3. Kelvin waves more North and South along coast from South America
4. Surface warming follows Kelvin Wave
5. Adjustment has a time scale, influences atmosphere-ocean dynamics and El Nino cycling