SMAST

Mixing in the Ocean

L. Goodman

General Physical Oceanography
MAR 555

School for Marine Sciences and Technology
Umass-Dartmouth

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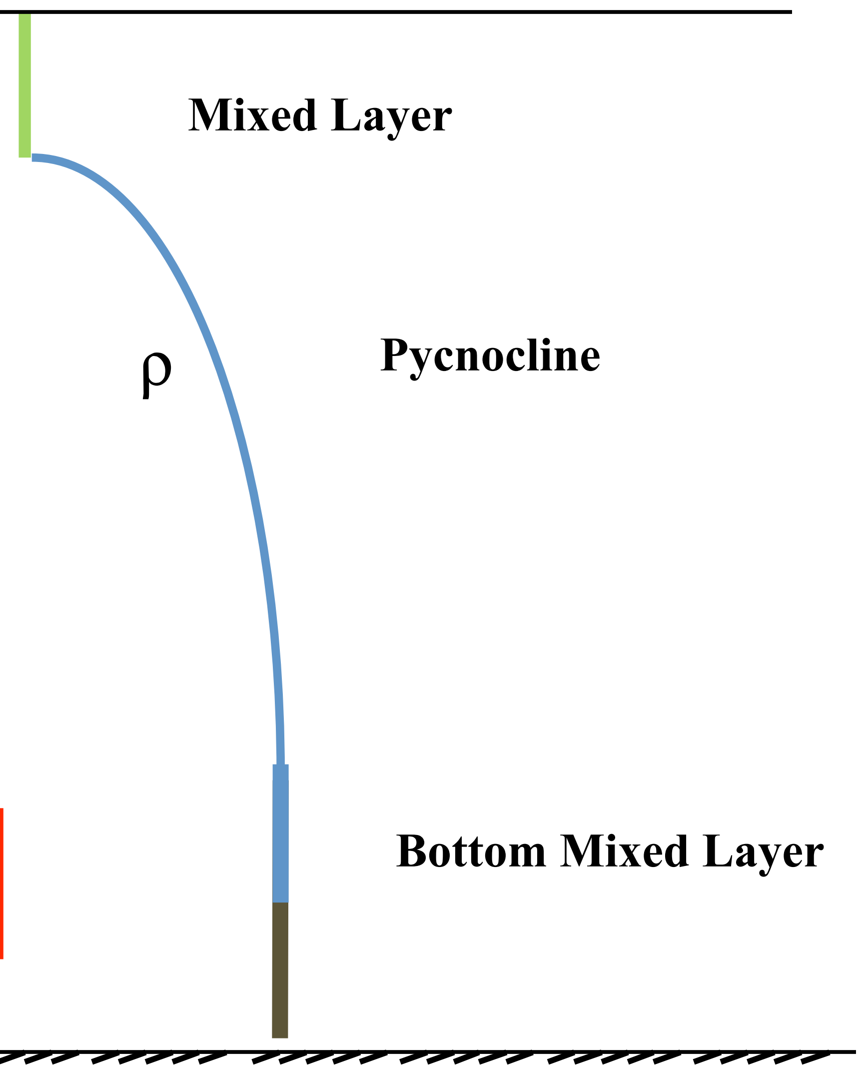
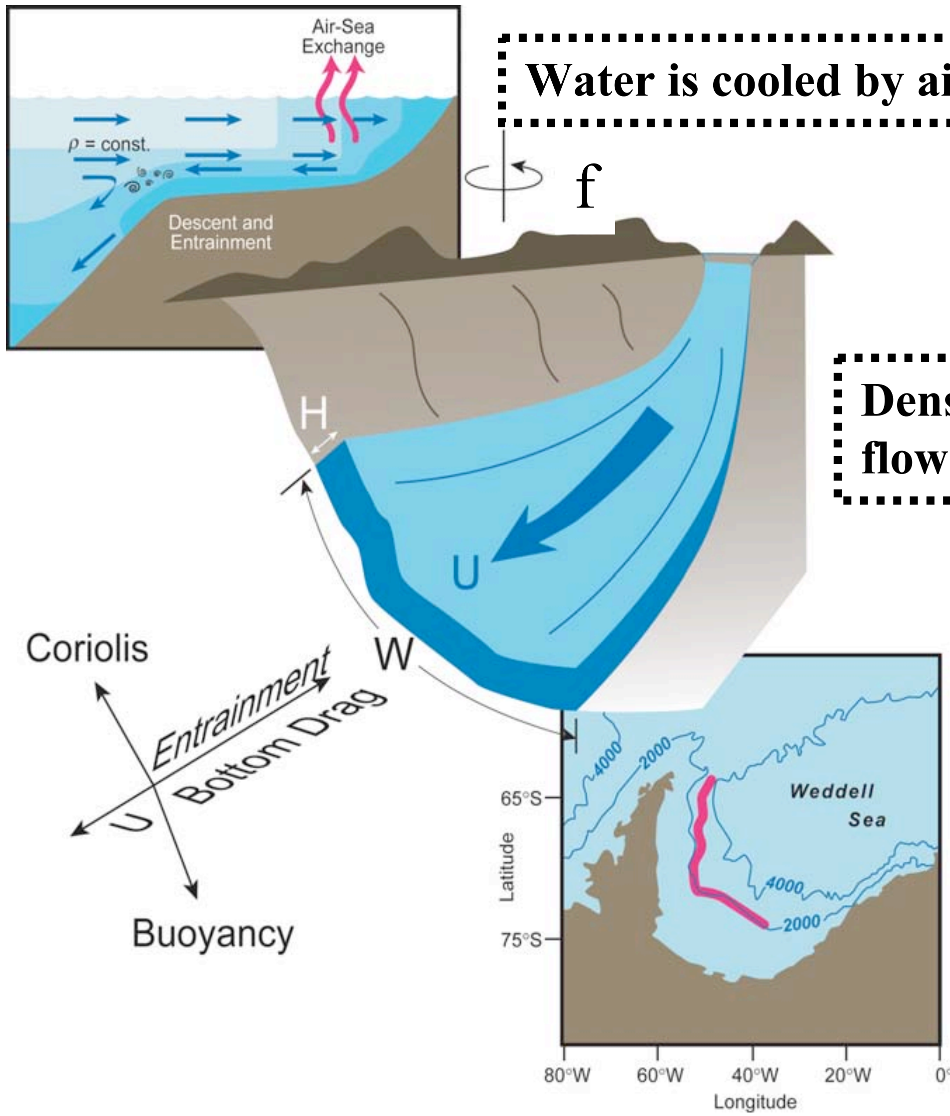


What gets mixed?

- Heat
- Salt
- Nutrients
- Oxygen
- Nitrogen
- Plankton
- Trace Metals
- Many others

How and why do they get mixed?
Why is it important?

Mixing in Water Mass Formation and Transport

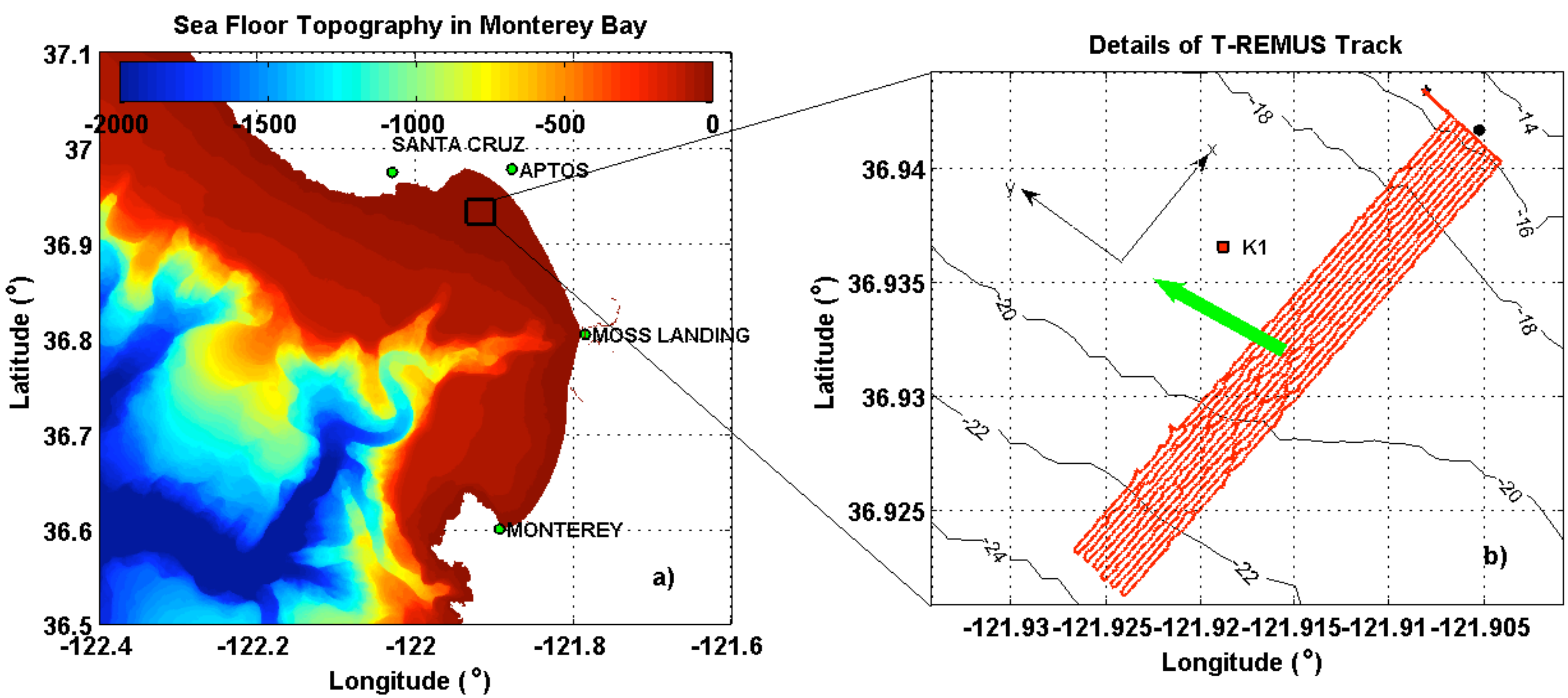


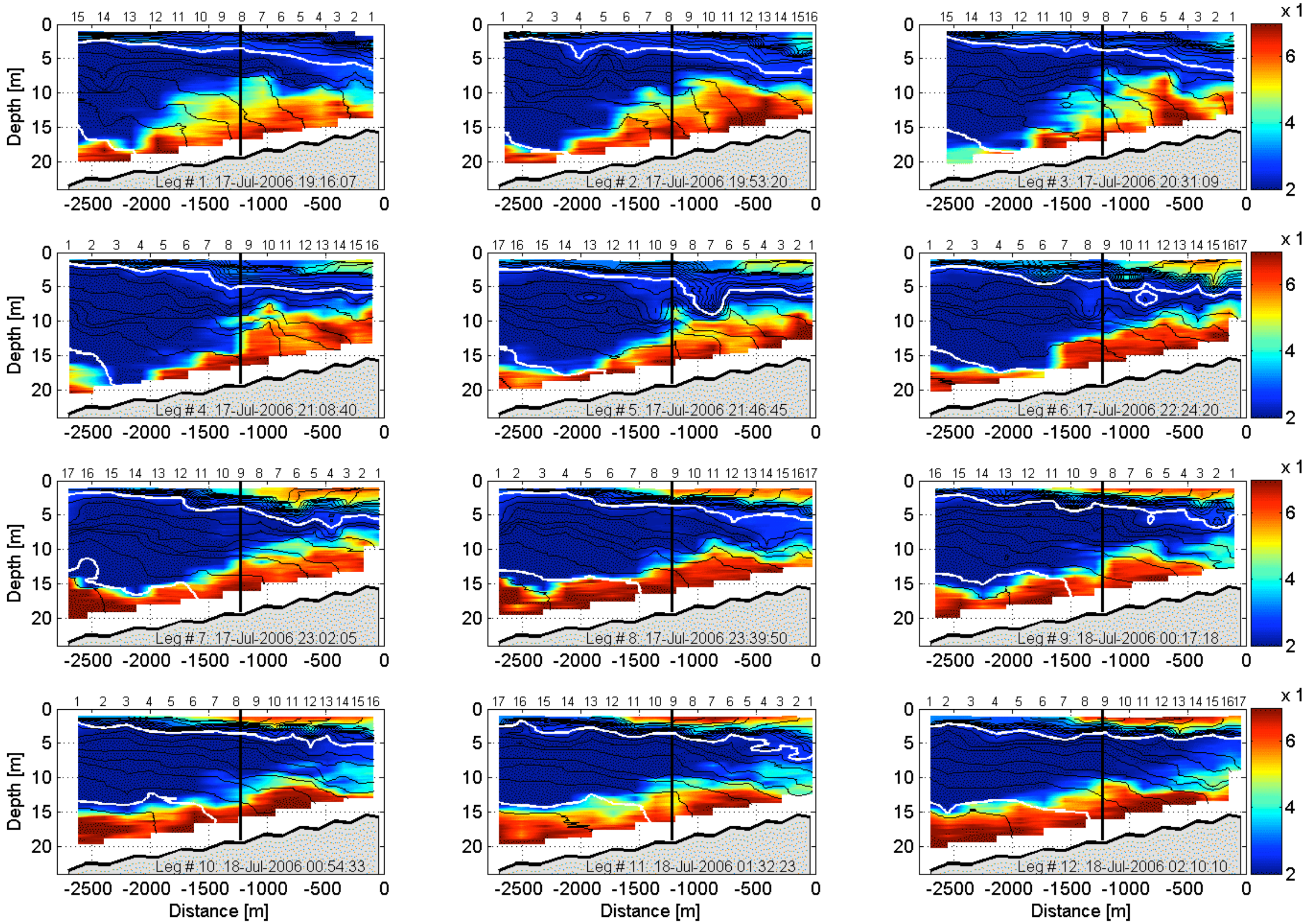
Water is cooled by air sea exchange

Dense water sinks and flows toward open ocean

Actual path near Antarctica

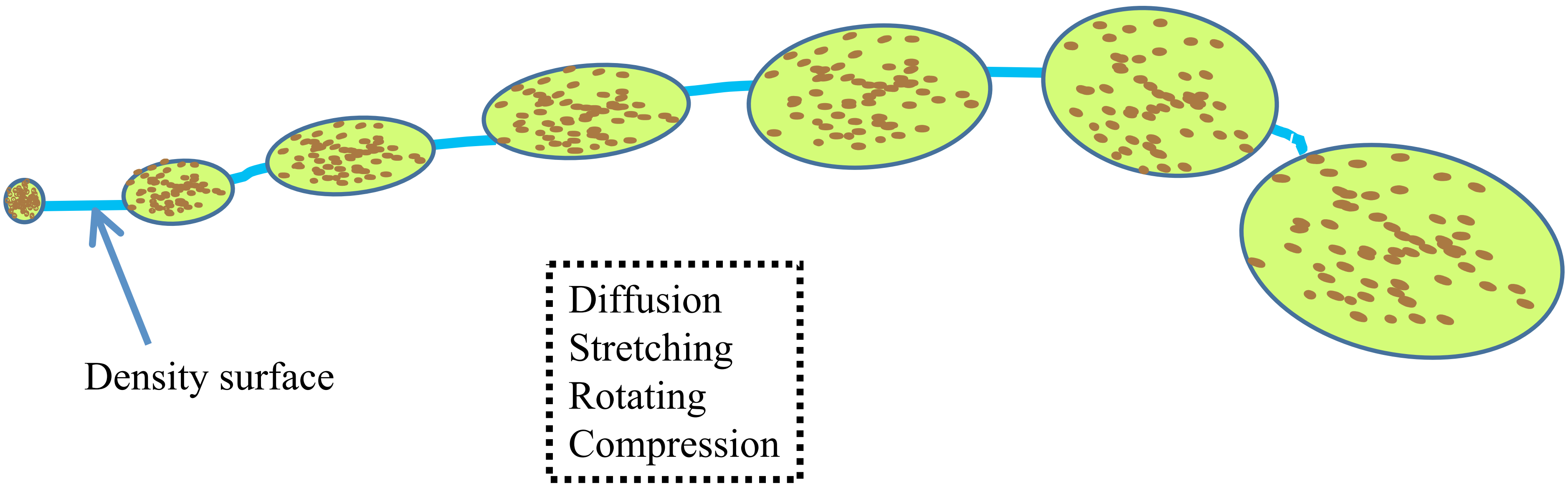
T-REMUS LOCO Deployment





T-REMUS observations during the LOCO 2006 experiment of Beta 700/Chlorophyll_a, a surrogate for bottom particulates. Figure shows bottom particles being swept up into the bottom mixed layer

Evolution of a patch of particles (plankton, detritus,others) In an evolving turbulent filed



Homework Question: Suppose particles of various sizes on the sea floor are swept up by the bottom flow into the turbulent bottom boundary layer. What size particles do you expect to remain the longest in the turbulent field and undergo diffusion? Explain.

Momentum Equation

$$\frac{\partial u_i}{\partial t} + \cancel{u_j \frac{\partial u_i}{\partial x_j}} - \cancel{(\vec{f} \times \vec{u})_i} = -\cancel{\frac{1}{\rho}\frac{\partial p}{\partial x_i}} - \cancel{g_i} + \nu \nabla^2 u_i$$

Scalar Evolution (T, S)

$$\frac{\partial \phi}{\partial t} + \cancel{u_j \frac{\partial \phi}{\partial x_j}} = k_\phi \nabla^2 \phi \qquad \phi = T, S$$

$\Rightarrow$

Diffusion Equation

$$\frac{\partial \phi}{\partial t} = k_\phi \nabla^2 \phi$$

$\nu = 10^{-6} \dfrac{m^2}{\sec}$ Molecular Diffusion of Momentum

$k_T = 1.4 \cdot 10^{-7} \dfrac{m^2}{\sec}$ Molecular Diffusion of Heat

$k_S = 1.4 \cdot 10^{-9} \dfrac{m^2}{\sec}$ Molecular Diffusion of Salt

Advective Diffusion Equation

$$\frac{\partial \phi}{\partial t} + u_i \frac{\partial \phi}{\partial x_i} = k_\phi \nabla^2 \phi$$

Example: Turbulent Diffusion of Heat

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = k_T \nabla^2 T$$

$$u_i = \bar{u}_i + u'_i$$

$$T = \bar{T} + T'$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{u}_j \frac{\partial \bar{T}}{\partial x_j} + \frac{\partial < u'_j T' >}{\partial x_j} = k_T \cancel{\nabla^2} T$$

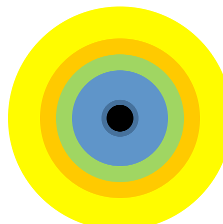
$$< u'_{x,y} T' > = \kappa_h \frac{\partial \bar{T}}{\partial (x, y)}$$ Horizontal Diffusion by Mesoscale Eddies

$$< w' T' > = \kappa_z \frac{\partial \bar{T}}{\partial z}$$ Vertical Diffusion by 3D Turbulence

Horizontal Advection **Vertical Advection** **Horizontal Diffusion** **Vertical Diffusion**

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \mathrm{v} \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = k_h \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + k_z \frac{\partial^2 T}{\partial z^2}$$

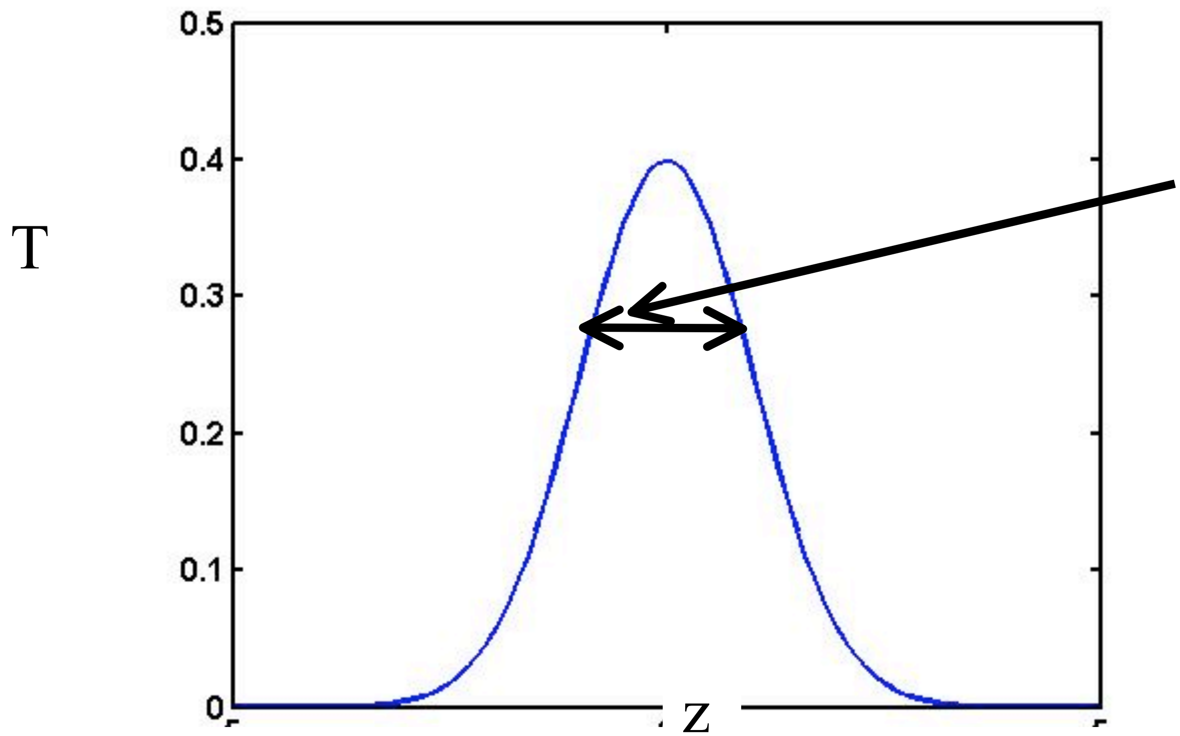
Example : 2D Diffusion



Example Vertical Diffusion

$$\frac{\partial T}{\partial t} = k_z \frac{\partial^2 T}{\partial z^2}$$

$$T = \frac{1}{\sqrt{4\pi k_z t}} \exp\left(-\frac{z^2}{4k_z t}\right)$$



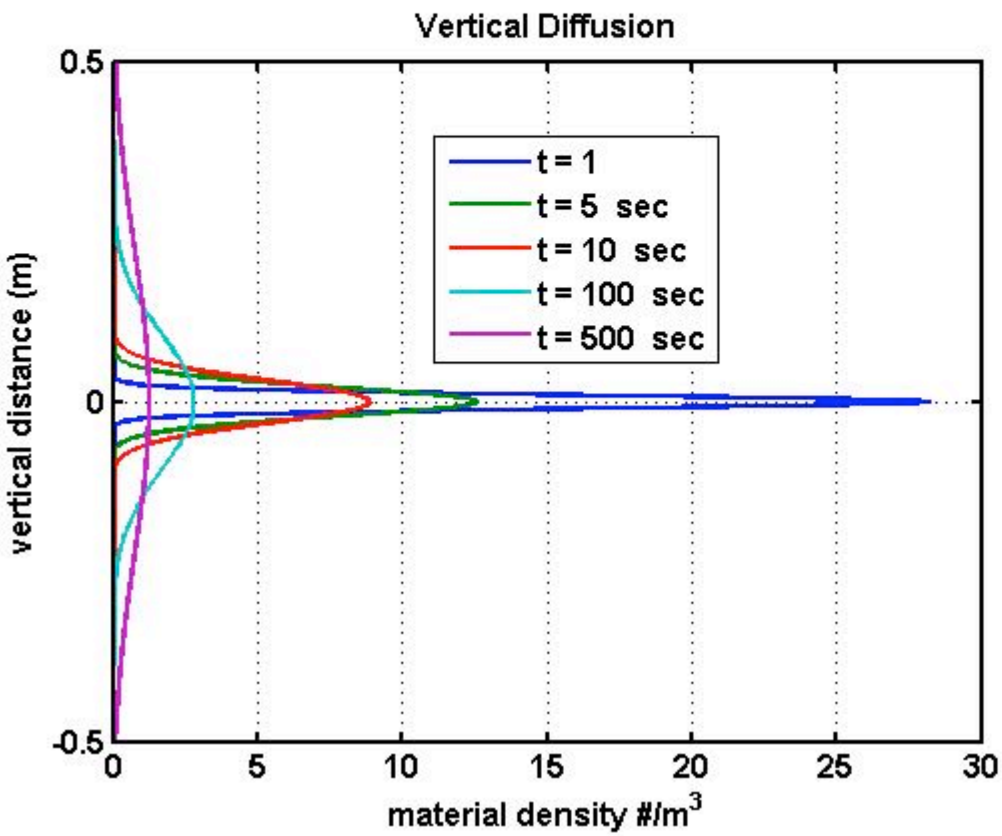
Variance

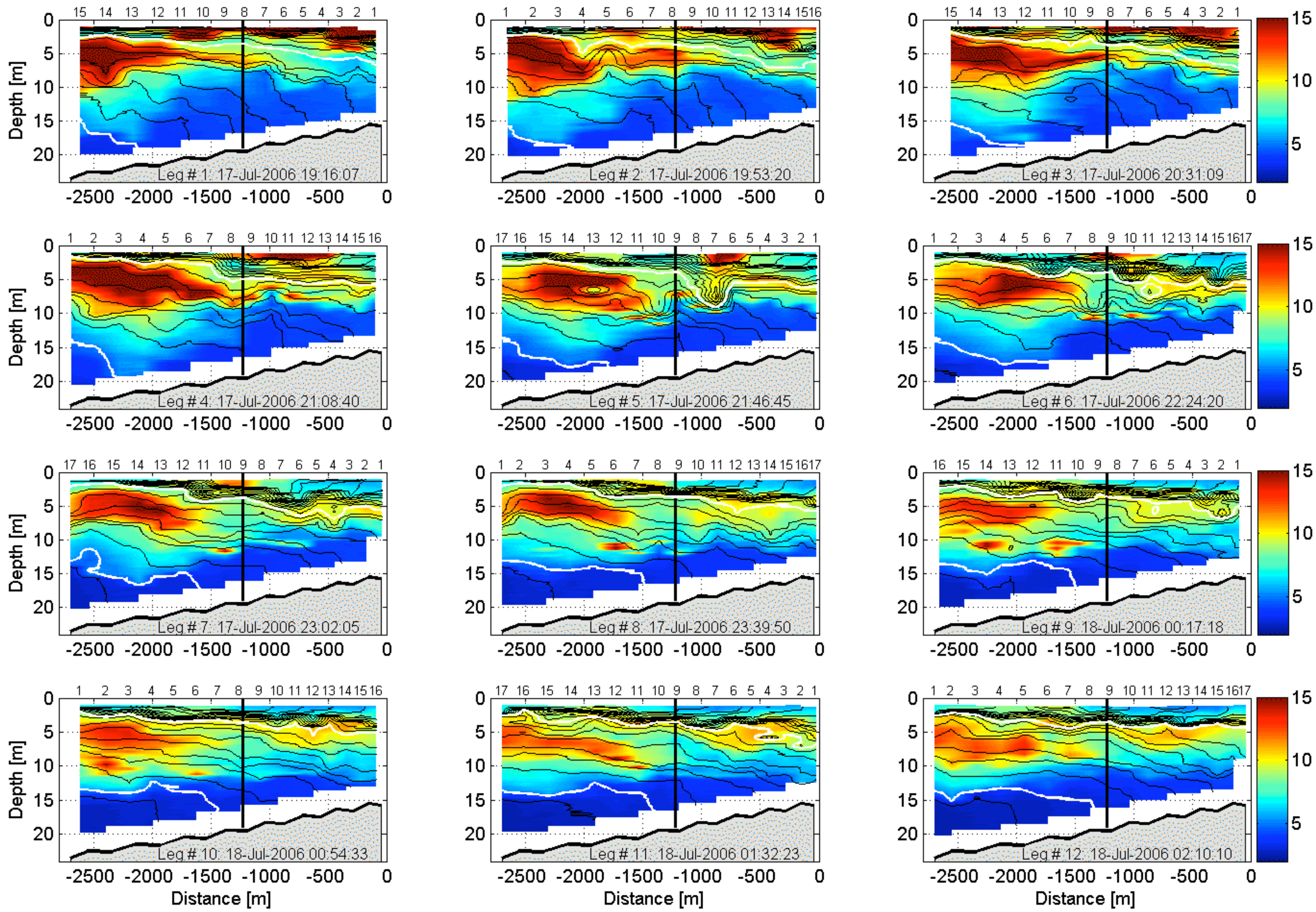
$$\sigma^2 = 2k_z t$$

Example Vertical Diffusion

$$\frac{\partial T}{\partial t} = k_z \frac{\partial^2 T}{\partial z^2}$$

$$k_z = 10^{-4} \frac{m^2}{\text{sec}}$$

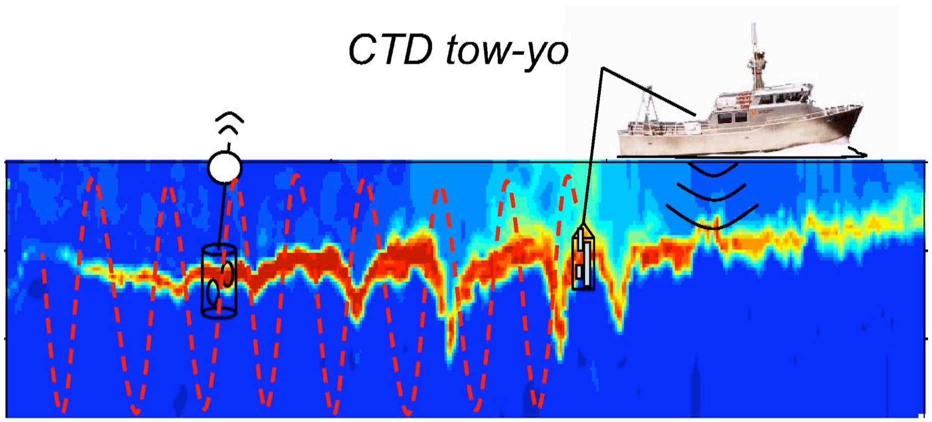




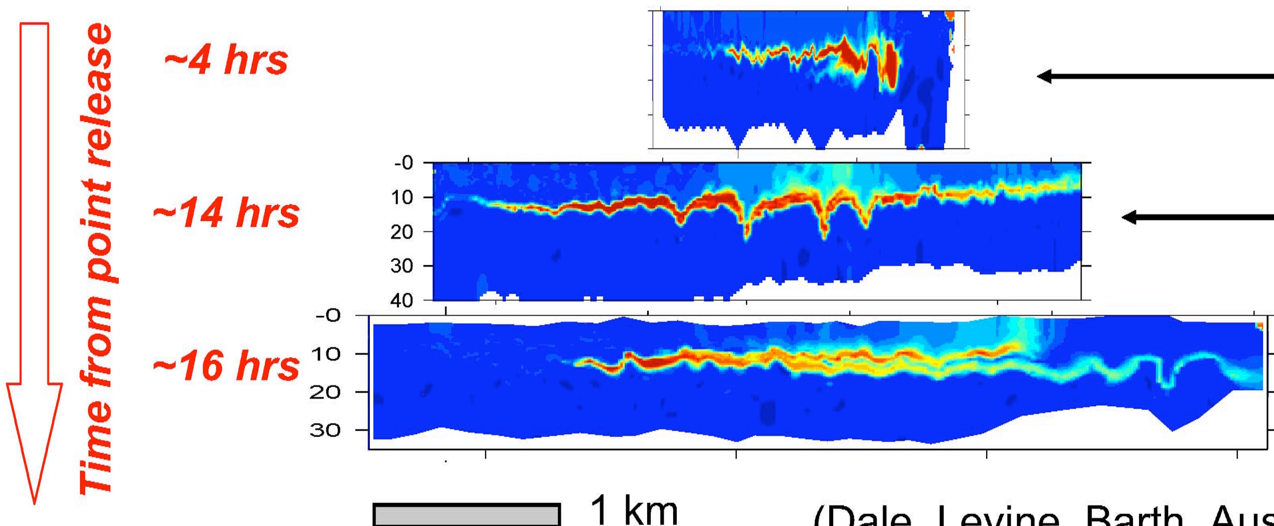
T-REMUS Observations of Chlorophyll a,a surrogate for phytoplankton, during the LOCO 2006 experiment (Wang and Goodman, 2008)

Cross-shelf dispersion and interleaving of a dye tracer on the Oregon shelf.

CTD tow-yo



Dye release



Rapid initial spreading

Internal wave effects

Interleaving thin layers

(Dale, Levine, Barth, Austin; GRL 2006)

General Case 3D Scalar Diffusion

$$\frac{\partial \phi}{\partial t} = k_h\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + k_z \frac{\partial^2 \phi}{\partial z^2}$$

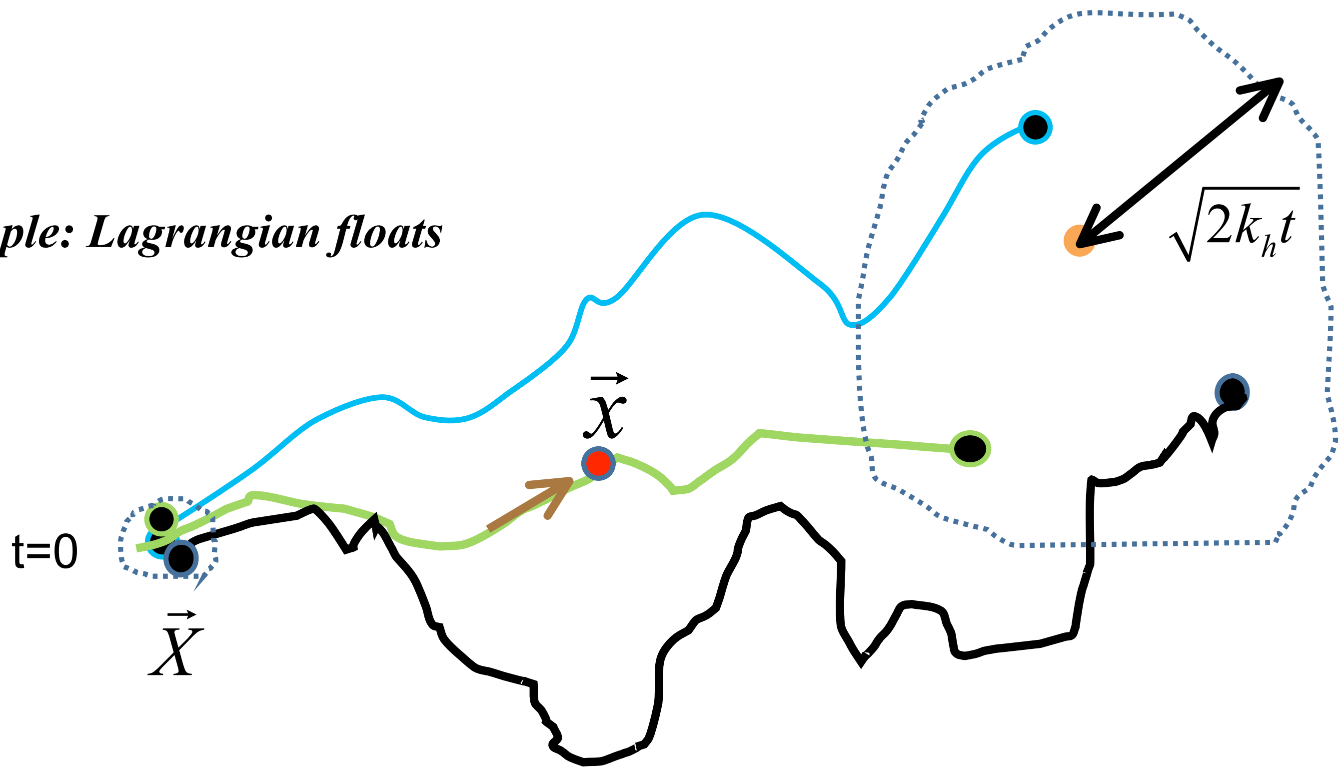
$$\phi = \frac{1}{\sqrt{(4\pi)^3 (k_h)^2 k_z t^3}} \exp - \left\{\frac{x^2 + y^2}{4k_h t} + \frac{z^2}{4k_z t}\right\}$$

$$\sigma_{x,y}{}^2 = 2k_{x,y} t \quad \sigma_z{}^2 = 2k_z t$$

Homework. A patch of plankton embedded in the flow field is found to in one day to diffuse horizontally 10 km over and vertically 5 m. Estimate the vertical and horizontal diffusivities. Estimate the characteristic velocity and space scales. What can you say about the motility velocity of the plankton? How much further would they diffuse horizontally and vertically in two days?

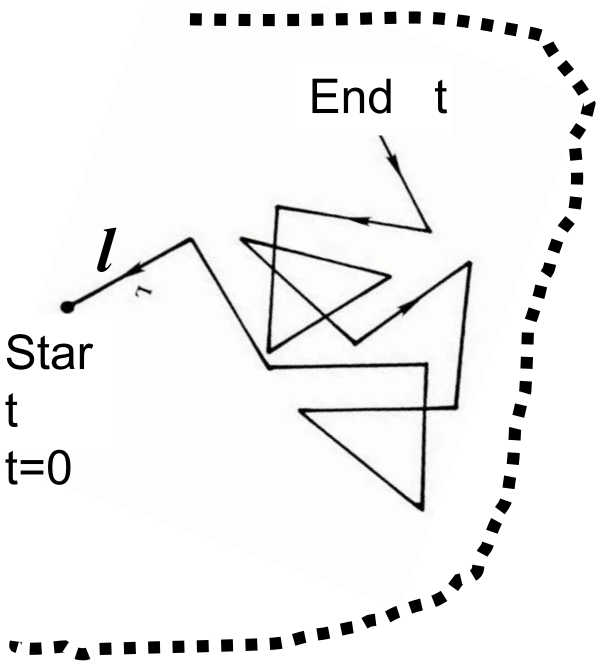
Lagrangian Approach to Diffusion

Example: Lagrangian floats



$$\vec{u} = \frac{d\vec{x}}{dt} = \lim \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} \Rightarrow \boxed{\vec{x} = \vec{X} + \int_0^t \vec{u}(X, t')dt'}$$

Turbulent Diffusion as a Random Walk Process



One dimensional Case

$$l \quad \overset{\longrightarrow}{\underset{\longleftarrow}{\text{or}}}$$

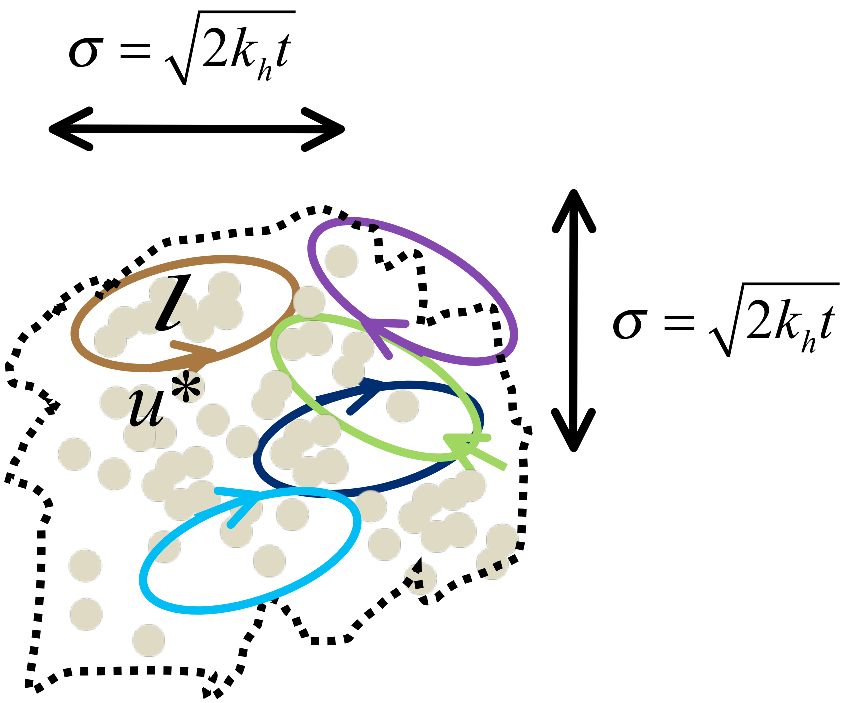
$$\boxed{\begin{array}{l} x = x_1 + x_2 + x_3 + \ldots x_n \quad \text{where } x_i = \pm l \\ < x_i x_j > \; = 0 \text{ for } i \neq j \text{ and } < x_i x_i > = l^2 \\ \overline{x_i} \; = \; < x_i > \; = 0 \end{array}}$$

$$\begin{aligned} < x^2 > \; &= \; < (x_1 + x_2 + x_3 + \ldots x_n)^2 > \\ &= \; < (x_1)^2 > + < (x_2)^2 > + < (x_3)^2 > + \ldots < (x_n)^2 > \\ &= \; n l^2 = \left\{\frac{l^2}{\Delta t}\right\} n \Delta t = \{2\mathrm{k}\}\ \mathrm{t} \end{aligned}$$

$$\Rightarrow 2k = \frac{l^2}{\Delta t} = (u^*)^2 \Delta t = u^* l$$

Interpret these results.
What is Δt ?

The Different Scales of a Turbulent Diffusive Process

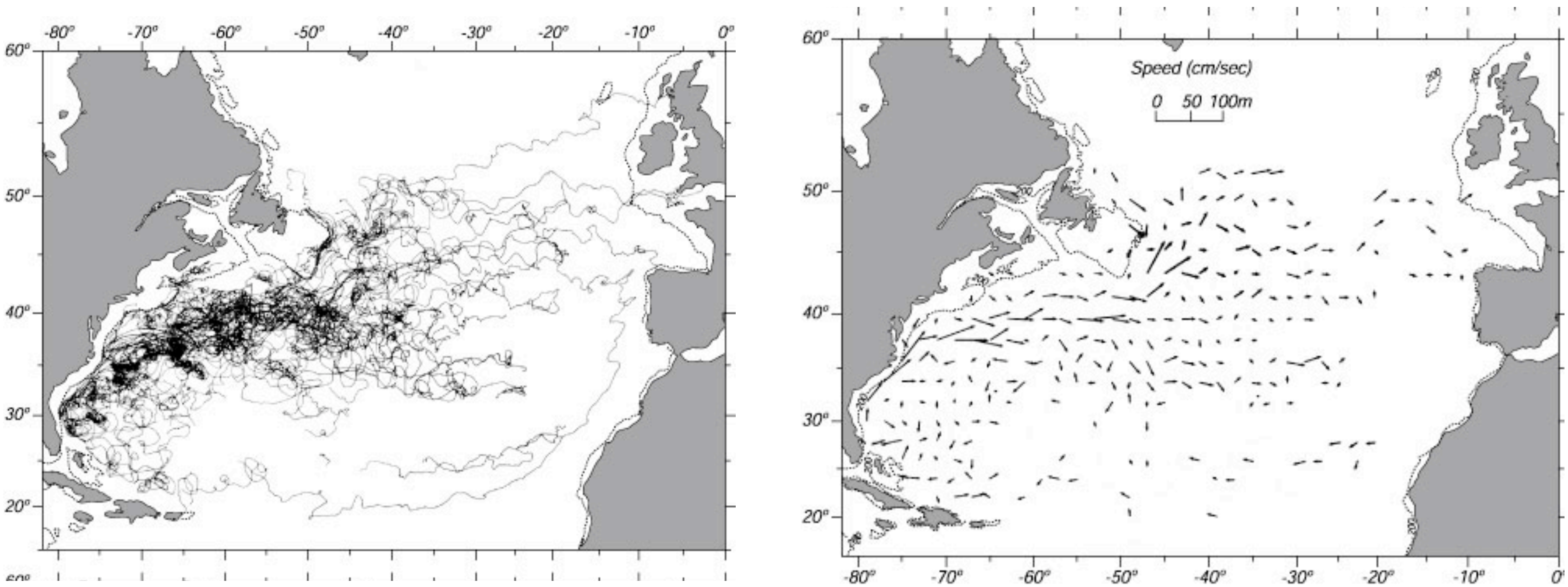


$$\sigma^2 = 2k_h t$$

$$2k_h = \frac{l^2}{\Delta t} = (u^*)^2 \Delta t = u^* l$$

Δt = Correlation Time
of the turbulent eddies

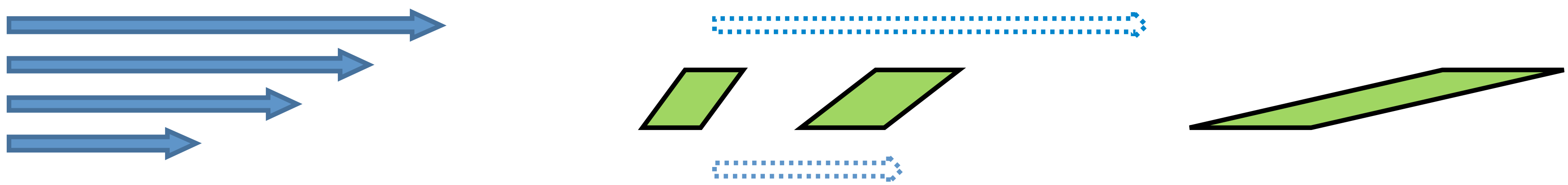
Homework: Suppose you release a large number of Lagrangian floats in a field of mesoscale eddies. You use a satellite tracking system tracking system which can only do one observation per day. If you know that these eddies are typically 50 km in horizontal extent with a decorrelation time of 10 days, how far would you expect the floats to diffuse in 100 days ? An SMAST student who has not yet taken physical oceanography tells you if you really want to impress your thesis adviser you should track these floats every day and every day tell your thesis adviser how far they have diffused. What do you think of that idea? *(Answering that you do not want to bother your thesis adviser is not acceptable!)*

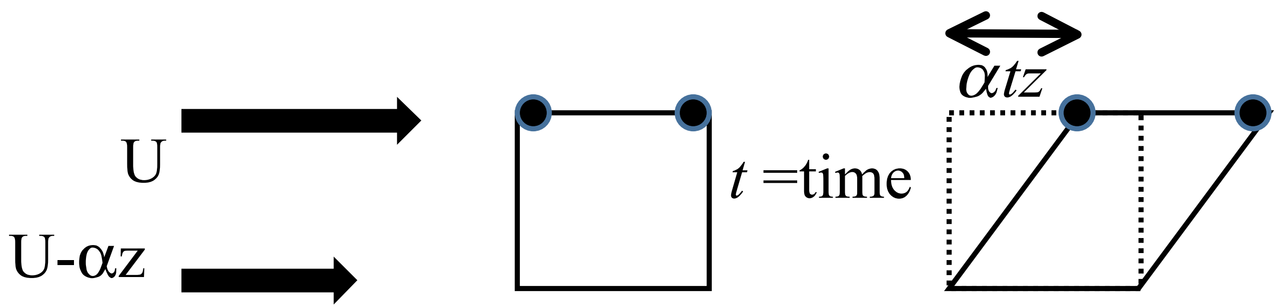


Left: Tracks of 110 drifting buoys deployed in the western North Atlantic.
Right: Mean velocity of currents in 2° × 2° boxes calculated from tracks above. Boxes with fewer than 40 observations were omitted. Length of arrow is proportional to speed. Maximum values are near 0.6 m/s in the Gulf Stream near 37°N 71°W. From Richardson (1981).

Role of Shear in Stretching and Dispersion

Current Shear





$L = \alpha t z$

L stretching distance

Shear Dispersion

k_z

$\sigma_z^{\,2} = 2k_z t$

$$\sigma_h^{\,2} = [1 + \frac{1}{12}(\alpha t)^2]\sigma_z^{\,2}$$

$$\Rightarrow k_h = [1 + \frac{1}{12}(\alpha t)^2]k_z$$

Steady State Diffusion
Munk's Model of the Permanent Thermocline

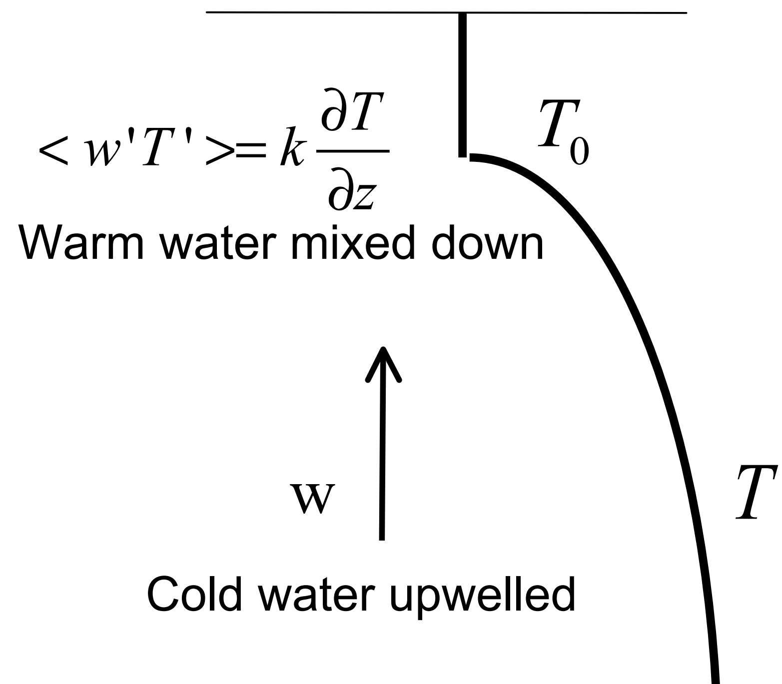
Vertical Temperature Equation

$$\cancel{\frac{\partial T}{\partial t}} + w\frac{\partial T}{\partial z} = k_z \frac{\partial^2 T}{\partial z^2}$$

$$w\frac{\partial T}{\partial z} - k_z \frac{\partial^2 T}{\partial z^2} = 0$$

$$T = T_o \exp(-\frac{z}{z_T})$$

$$z_T = \frac{k_z}{w}$$

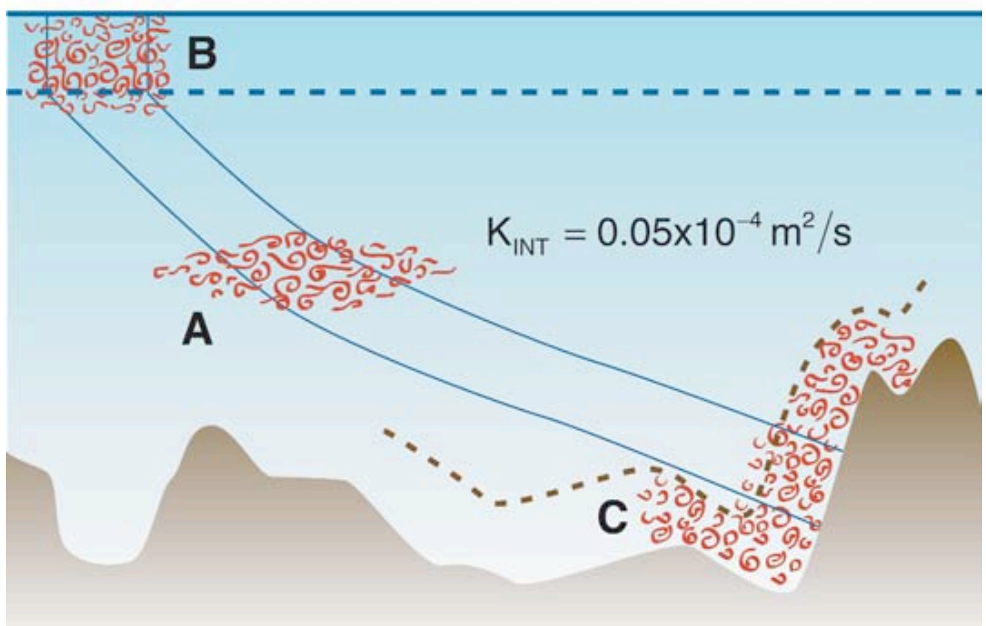


From measurements of the permanent thermocline
and from large scale thermohaline balance

$$z_T = 1\ km\ ,\ w \approx 3\frac{cm}{Year} \Rightarrow k_z \approx 10^{-4}\frac{m^2}{\sec}$$

But microstructure (turbulence) measurements suggest $k_z < 10^{-5}\frac{m^2}{\sec}$ **!!**

Three possible paradigms for abysal ocean mixing
(Kunze and Smith, 2004)



The longstanding hypothesis of uniformly distributed mixing in the ocean interior (A) requires turbulent eddy diffusivities $k_z \approx 10^{-4}\,\frac{m^2}{sec}$ while measured values are only $k_z \approx .05 ?\, 10^{-4}\,\frac{m^2}{sec}$.

Alternatives are either surface-enhanced mixing where density surfaces outcrop at polar latitudes (B) or bottom enhanced mixing over rough topography (C), the products of which then stir along density surfaces to fill the interior.

Homework: A patch of dye initially 5 m vertically by 100 m horizontally is located in a constant vertical velocity shear of $\alpha = .01 \ \sec^{-1}$. Microstructure turbulence measurements in this region suggest a vertical diffusivity of $k_z = 10^{-4} \ \frac{\mathrm{m}^2}{\sec}$.

How far does the patch disperse horizontally and vertically in 1 second, 1 minute, 1 hour, 1 day? When does shear dispersion effects become more important than the laminar stretching effects in determining horizontal structure.?

Homework: In a certain region on the continental shelf suppose there is a surface input of heat of F =200 Watts/m^2. The turbulent diffusivity in this region is estimated as 10^{-4} m^2/sec. If there is no upwelling what is the temperature gradient in the water.

If there was an upwelling velocity of w = .1 mm/sec what would the temperature gradient profile be. What is the buoyancy profile?

$$F = \text{Heat Flux} = \rho c < w'T' > \quad (\text{units of } \frac{Watts}{m^2})$$

$$\rho = \text{Seawater Density} = 10^3 \, \frac{kg}{m^3}$$

$$c = \text{Seawater Specific Heat} = 4.2 \times 10^3 \, \frac{J}{kg\text{ - }^oC}$$