Ekman Dynamics

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MAR 555

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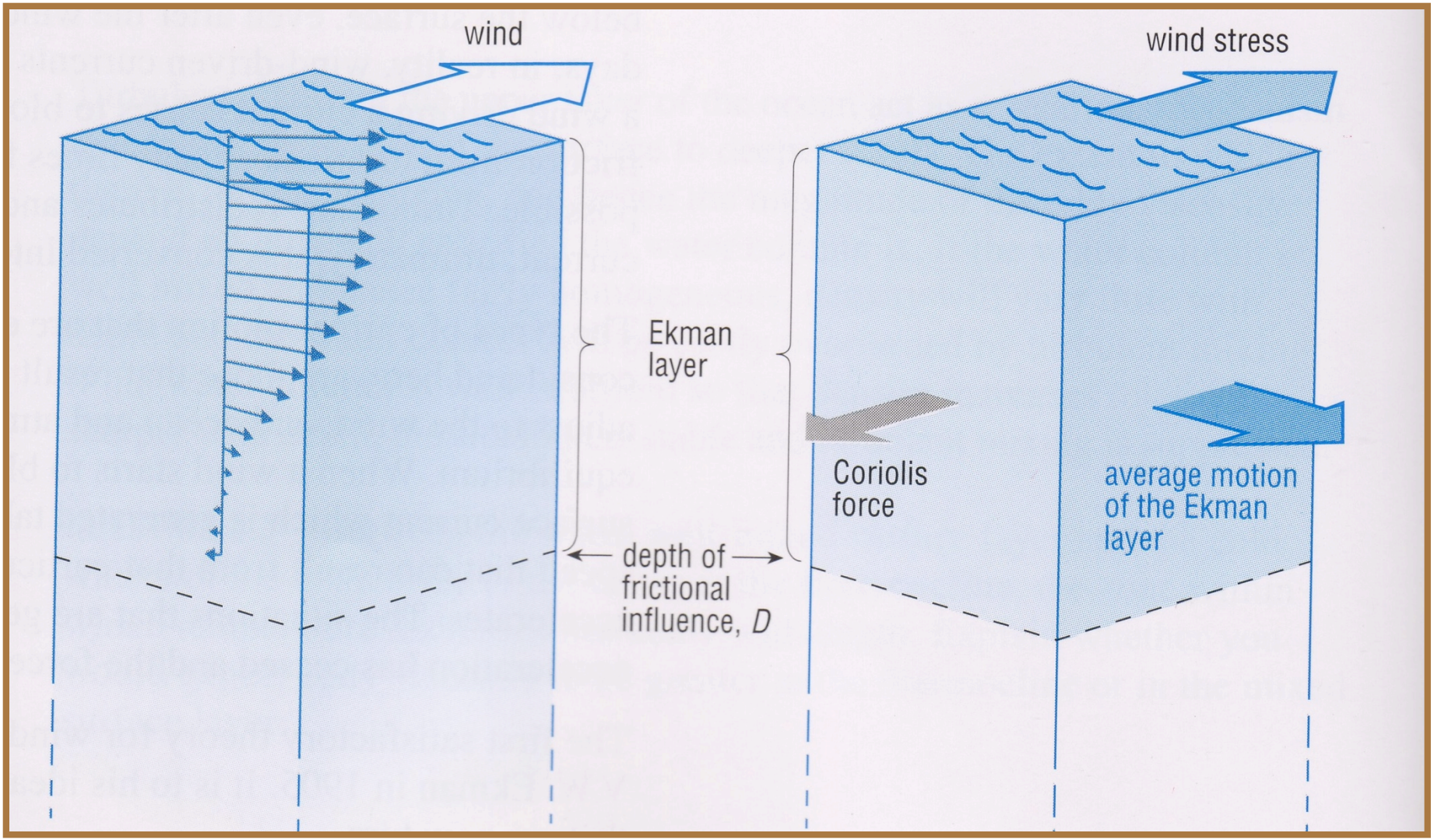
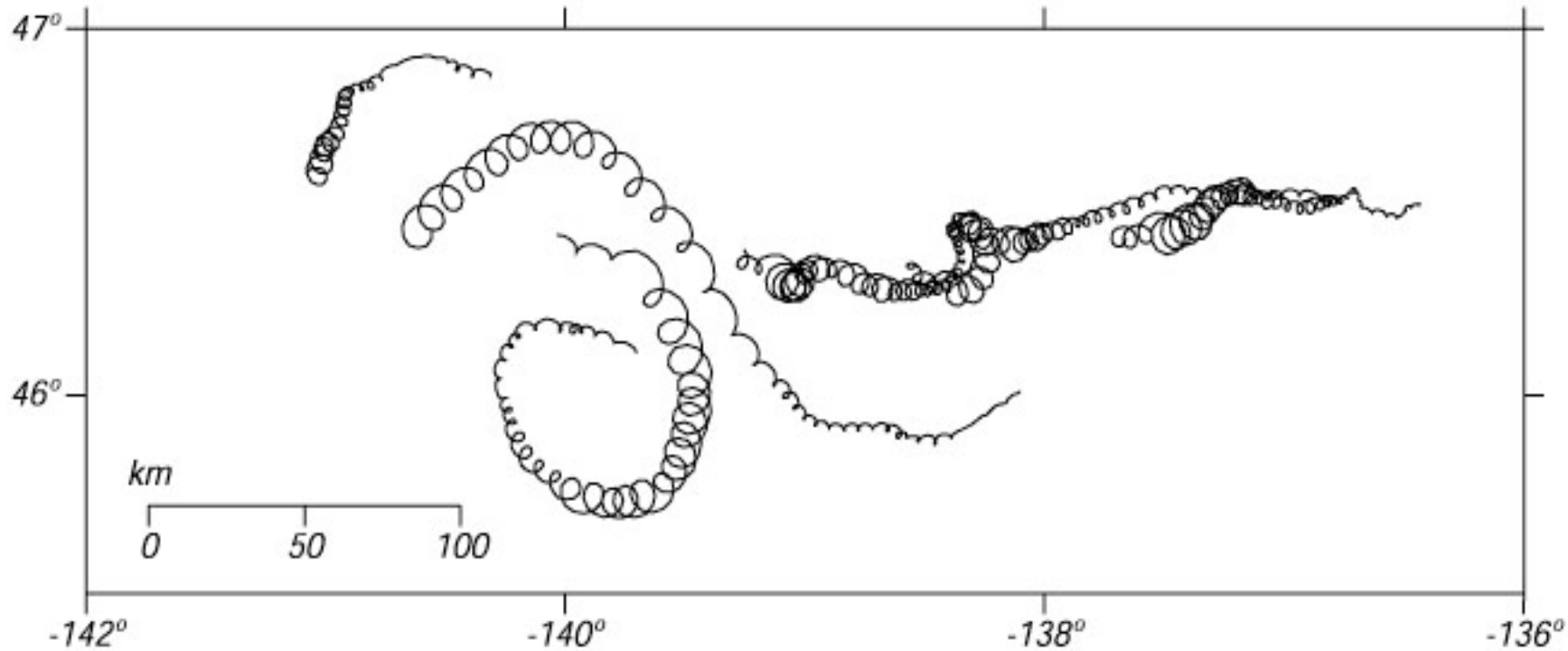
Ekman Dynamics I (IPO-9)



Inertial Oscillations



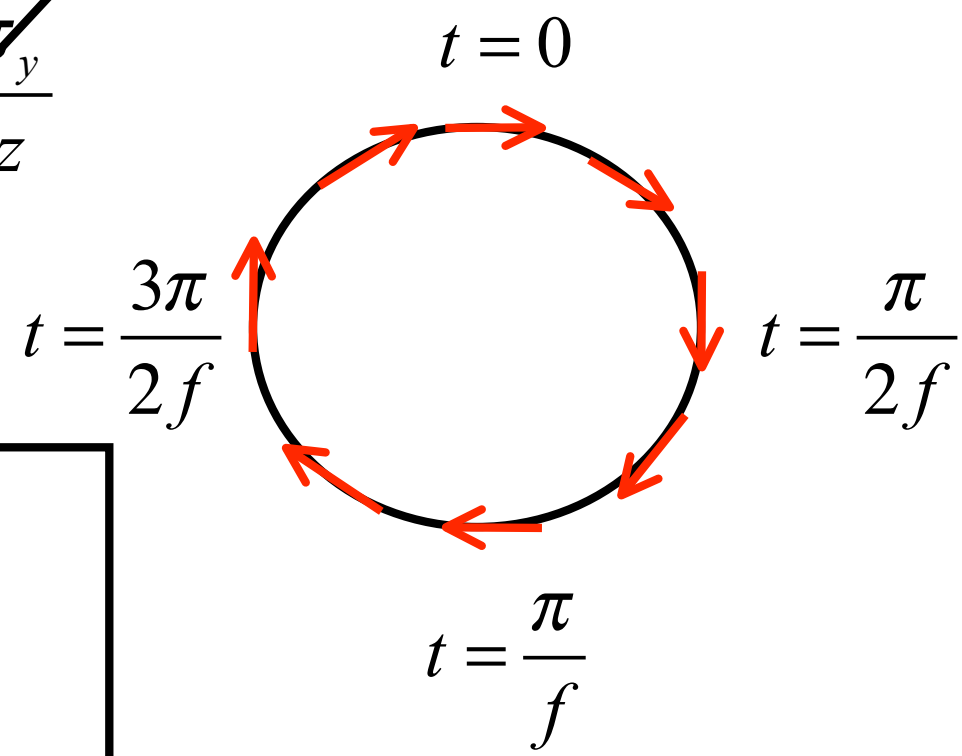
IPO Figure 9.1 Inertial currents in the North Pacific in October 1987 (days 275-300) measured by holey-sock drifting buoys drogued at a depth of 15 meters. Positions were observed 10-12 times per day by the Argos system on NOAA polar-orbiting weather satellites and interpolated to positions every three hours. The largest currents were generated by a storm on day 277. Note: these are not individual eddies. The entire surface is rotating. A drogue placed anywhere in the region would have the same circular motion. From van Meurs (1998).

Inertial Oscillations

$$\frac{Du}{Dt} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$\frac{D\mathrm{v}}{Dt} + fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

$$\frac{D}{Dt} = u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y}$$

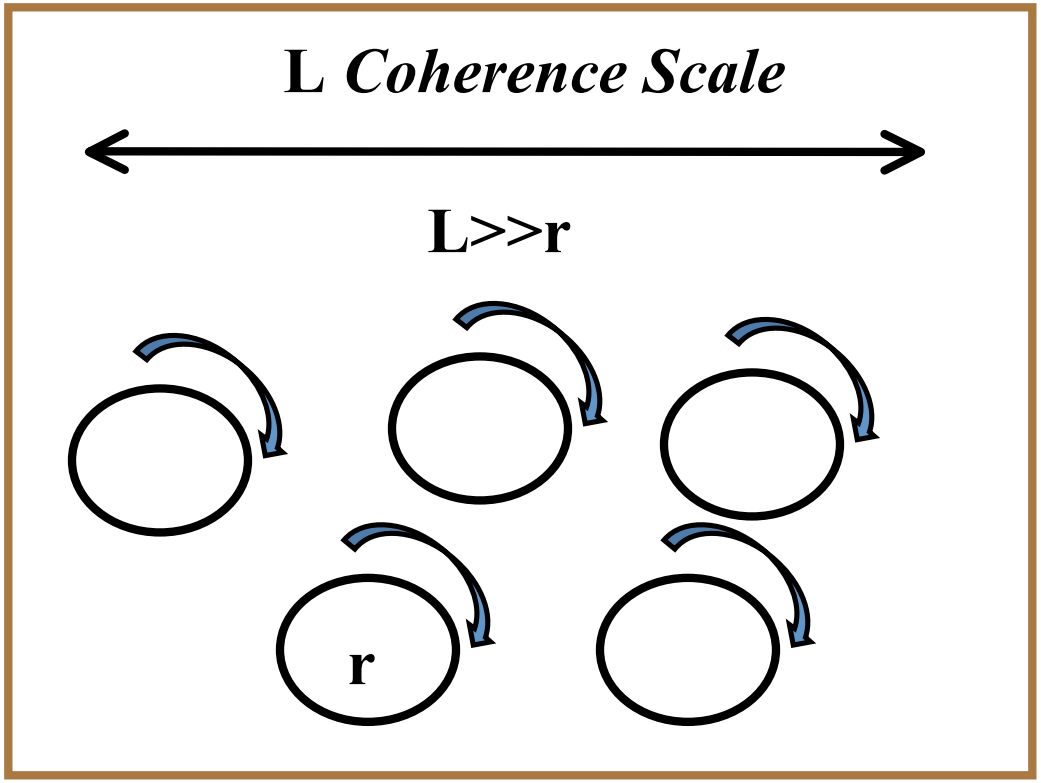


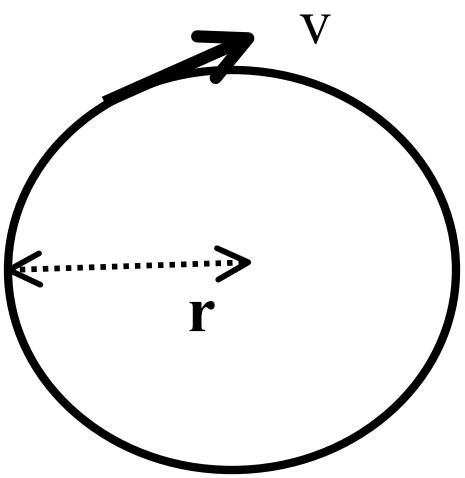
Solution

$$u = U\cos(ft)$$

$$\mathrm{v} = U\sin(ft)$$

Inertial Oscillations





$$v = rf$$

Homework: The wind suddenly stops blowing and sets up inertial oscillations in the upper ocean. You place a float in the upper ocean and find that it moves around in a circle of radius 10 km. If you are located at 45 degrees N how long will it take the float to do a complete cycle? What is the speed which the floats move ? Suppose you deployed 10 such floats describe how each one would move relative to the others.

Ekman Dynamics

$$\cancel{\frac{Du}{Dt}} - \cancel{fv} = -\frac{1}{\rho}\cancel{\left(\frac{\partial p}{\partial x}\right)} + \cancel{\frac{1}{\rho}\frac{\partial \tau_x}{\partial z}}$$

$$\cancel{\frac{Dv}{Dt}} + fu = -\frac{1}{\rho}\cancel{\left(\frac{\partial p}{\partial y}\right)} + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

Example A: Steady state, no pressure gradient, wind blows in "y" direction, *constant density*

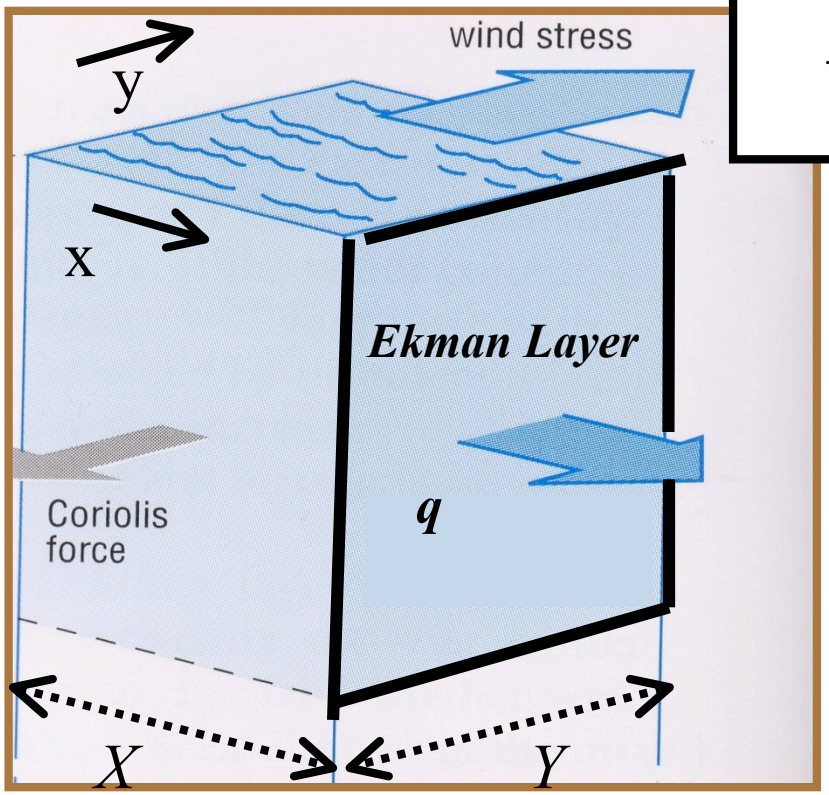
$$fu = \frac{1}{\rho_0}\frac{\partial}{\partial z}\tau_y$$

$$q = Y\int_{-D}^{0} u\,dz = \frac{Y}{f}\frac{\{\tau_y\}_{z=0}}{\rho_0}, \textit{where } Y \textit{ is length over which the stress occurs.}$$

$\boxed{q = \textit{Volume Transport}}$ $\boxed{M = \rho q \;=\; \textit{mass or Ekman transport}}$

Note: D is defined by $\{\tau_y\}_{z=D} = 0$

Ekman Transport



$$\{\tau_y\}_{z=0}$$

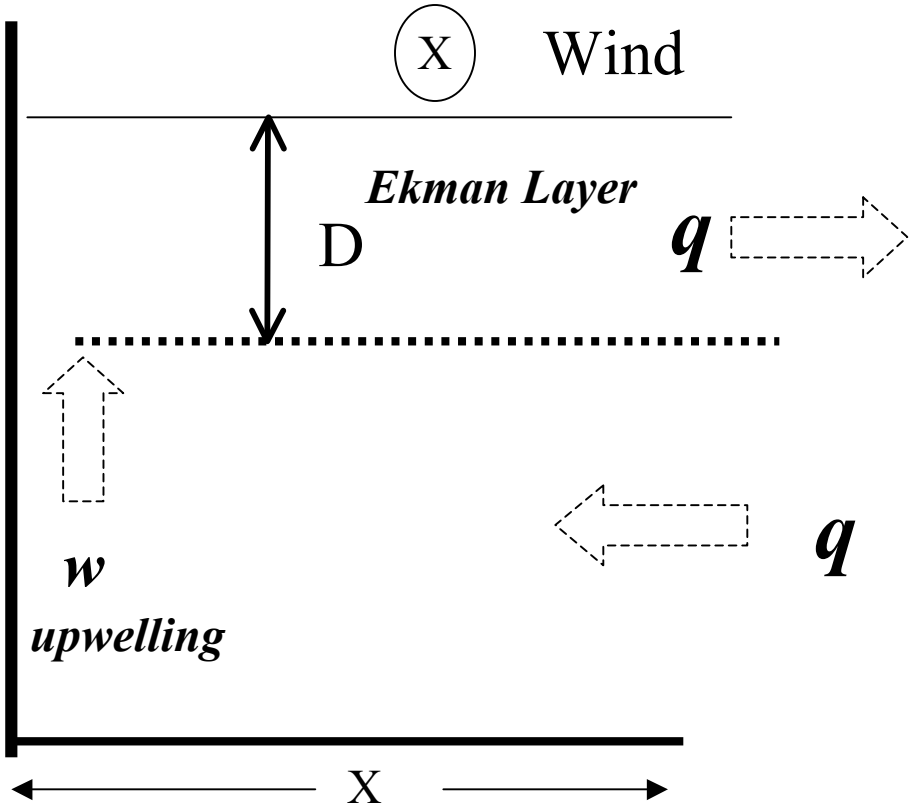
$$q = \text{Volume Transport}$$

$$= Y\int u\,dz = \frac{Y}{f}\frac{\{\tau_y\}_{z=0}}{\rho_0}$$

$$= \frac{Y}{f}(u^*)^2$$

Note:
X,Y are the characteristic lengths over which the wind stress occurs and transport occur.

Coastal Upwelling

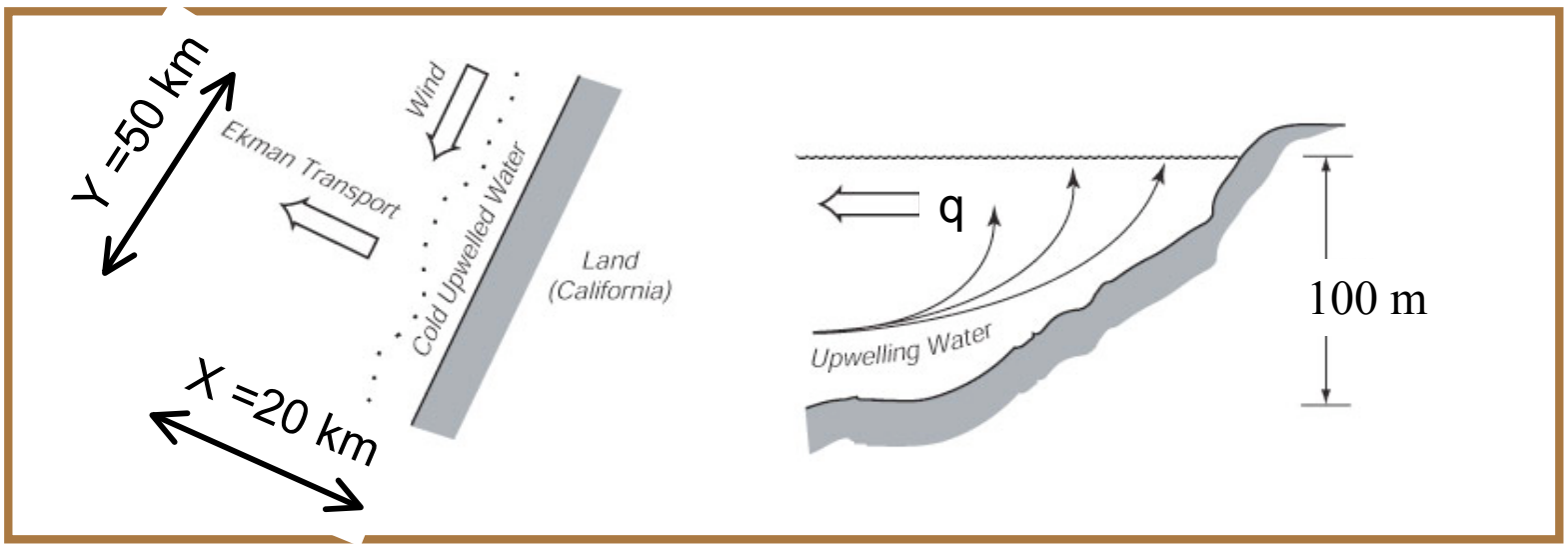


$$\frac{\partial u}{\partial x} = -\frac{\partial w}{\partial z} \Rightarrow \boxed{w = -\int \frac{\partial u}{\partial x} dz \approx -\frac{(-q)}{XY} = \frac{(u^*)^2}{Xf}}$$

Upwelling Example

Wind (U_{10}) blows to the south off of Northern California just north of Monterey Bay (latitude 37°N) at steady rate of 10 m/sec. If the coastal region being affected has dimensions , X = 20 km, Y = 50 km, as shown in the figure below, estimate :

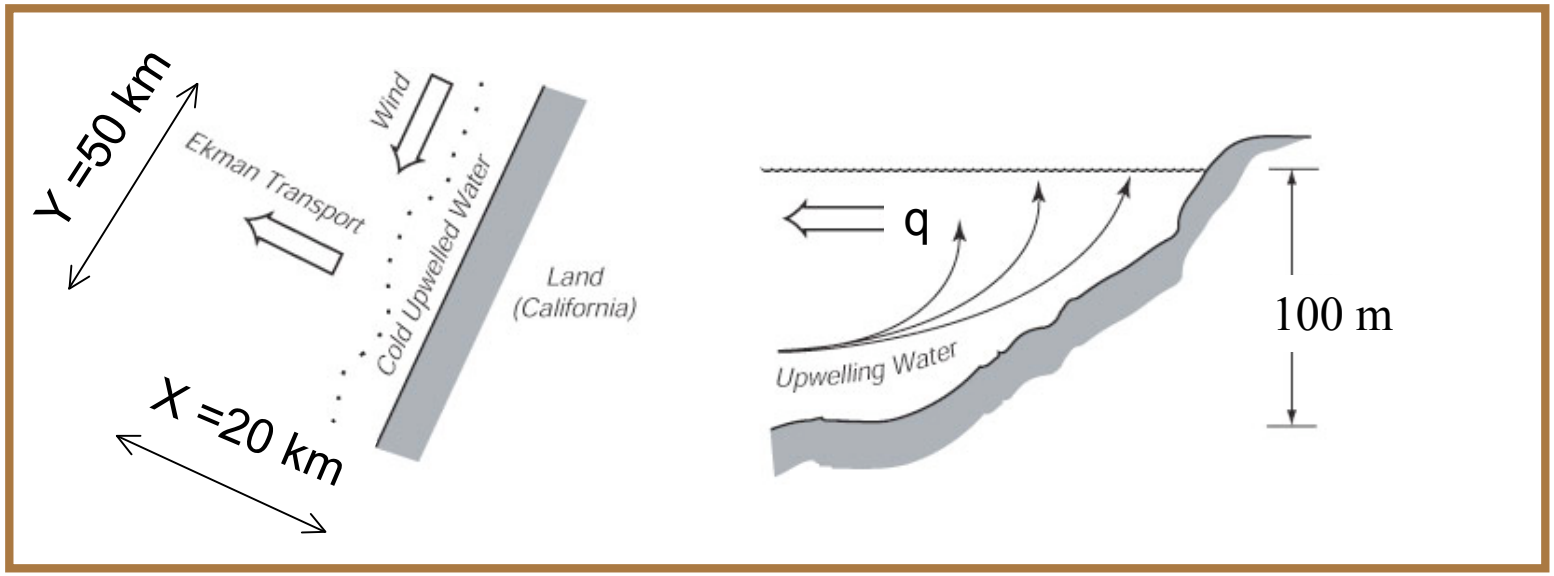
(a)the Ekman volume transport , q , (b) upwelling velocity, w, and (c) how long it would take nutrients located at a depth of 100 m to rise to the surface.



Note: $f = 2*\Omega*\sin(\theta) = 2*\dfrac{2*\pi}{24*3600}*\sin(37*\dfrac{\pi}{180})\ \sec^{-1} = 8.8\cdot 10^{-5}\ \sec^{-1}$

$$(u^*)^2 = \frac{\tau}{\rho_{water}} = \frac{\rho_{air} C_D U_{10}{}^2}{\rho_{water}} = \frac{1\dfrac{kg}{m^3}\cdot 2.5\cdot 10^{-3}\cdot (10\dfrac{m}{\sec})^2}{1000\dfrac{kg}{m^3}} = 2.5\cdot 10^{-4}\ (\frac{m}{\sec})^2$$

Upwelling Example Continued



$$\text{(a) } q = \frac{Y}{f}(u^*)^2 = \frac{5\cdot 10^4 m}{8.8\cdot 10^{-5}\,\text{sec}^{-1}} 2.5\cdot 10^{-4}\ (\frac{m}{\text{sec}})^2 = 1.42\cdot 10^5\,\frac{m^3}{\text{sec}} = .142 Sv$$

$$(b)\ w = \frac{q}{XY} = \frac{(u^*)^2}{Xf} = \frac{2.5\cdot 10^{-4}\ (\frac{m}{\text{sec}})^2}{2.0\cdot 10^4 m\cdot 8.8\cdot 10^{-5}\,\text{sec}^{-1}} = 1.4\cdot 10^{-4}\,\frac{\text{m}}{\text{sec}} = 12\,\frac{\text{m}}{\text{day}}$$

$$(c)\ \ t = \frac{H}{w} = \frac{100m}{12\,\frac{m}{day}} = 8\ days$$

Ekman Dynamics: Upper Ocean

(*Assumptions: steady state, no surface slope, p=0, infinitely deep water, constant density* $\rho = \rho_0$)

Eddy Viscosity assumption

$$-f\mathrm{v} = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \qquad \tau_x = \rho_0 k \frac{\partial u}{\partial z}$$

$$+$$

$$fu = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \qquad \tau_y = \rho_0 k \frac{\partial \mathrm{v}}{\partial z}$$

Solution ***let (z → -z), z positive downward***

$$u = U\exp[-bz]\sin(bz + \frac{\pi}{4}) \qquad b = \sqrt{\frac{f}{2k}}$$

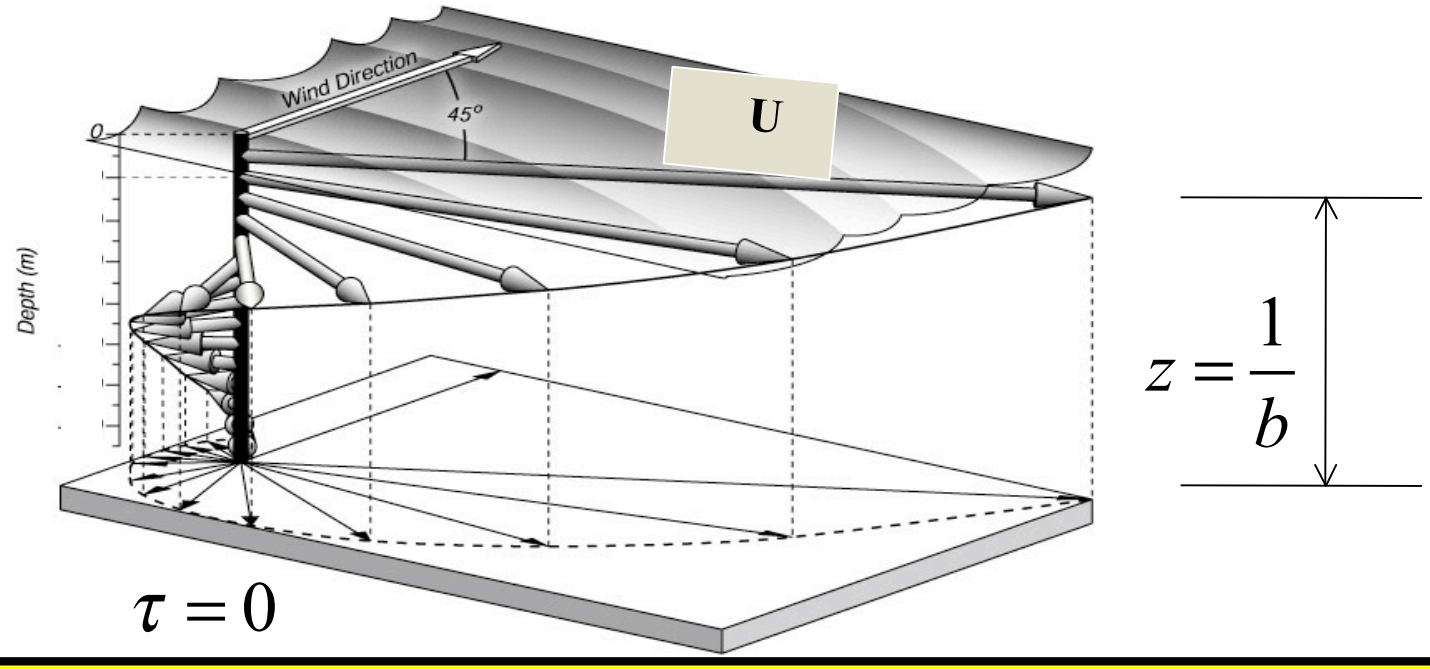
$$\mathrm{v} = U\exp[-bz]\cos(bz + \frac{\pi}{4})$$

Note: b^{-1} is the "effective" Ekman depth where $|\vec{u}| = e^{-1}U = .37U$

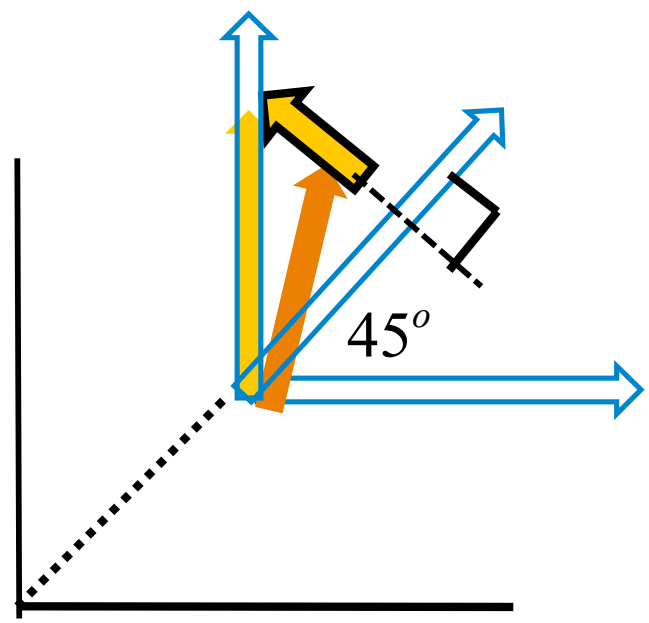
$$U = \frac{\tau_s}{\rho_0\sqrt{fk}} = \frac{(u^*)^2}{\sqrt{fk}}$$

$bH >> 1$ (Deep water assumption)

$$\tau_s = \rho_0 k \frac{\partial u}{\partial z}\Big|_{z=0} = \rho_0 (u^*)^2$$

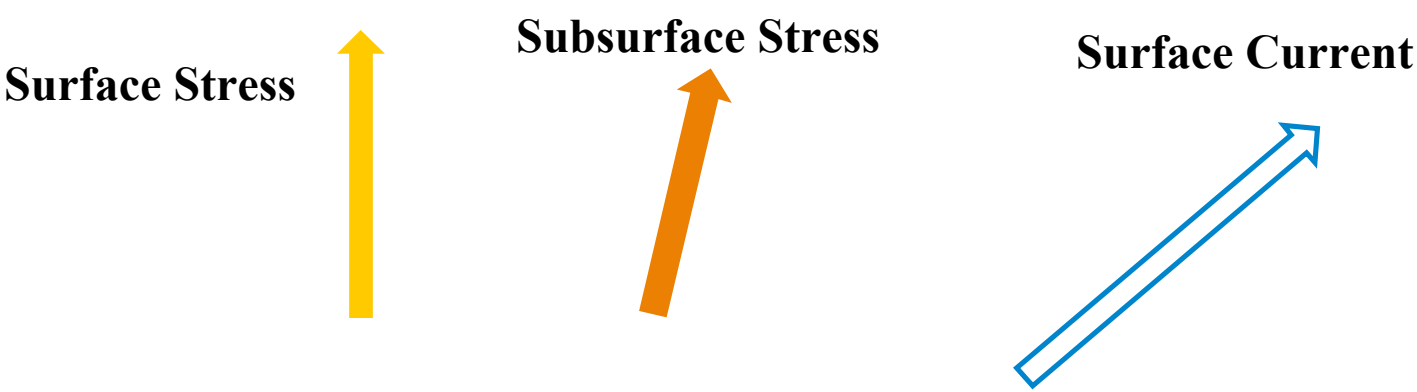


Homework: Wind, U_{10}, blows across the outer edge of the RI shelf to the east at 10 m/sec over a cross shelf extent of 50 km. CODAR measurements indicate a surface current magnitude of .1 m/sec, assuming that the surface pressure gradient is very small, (a) What is the direction of the surface current? (b) What is the value of eddy viscosity?(c) What is the effective depth of the Ekman layer? (d) What is the magnitude and direction of the current field at 2 m below the surface, at 10 m below the surface? (e) What is your estimate of the turbulent velocity at the surface, at depths of 2, 10 meters, on the bottom ? (f) Estimate the upwelling velocity.

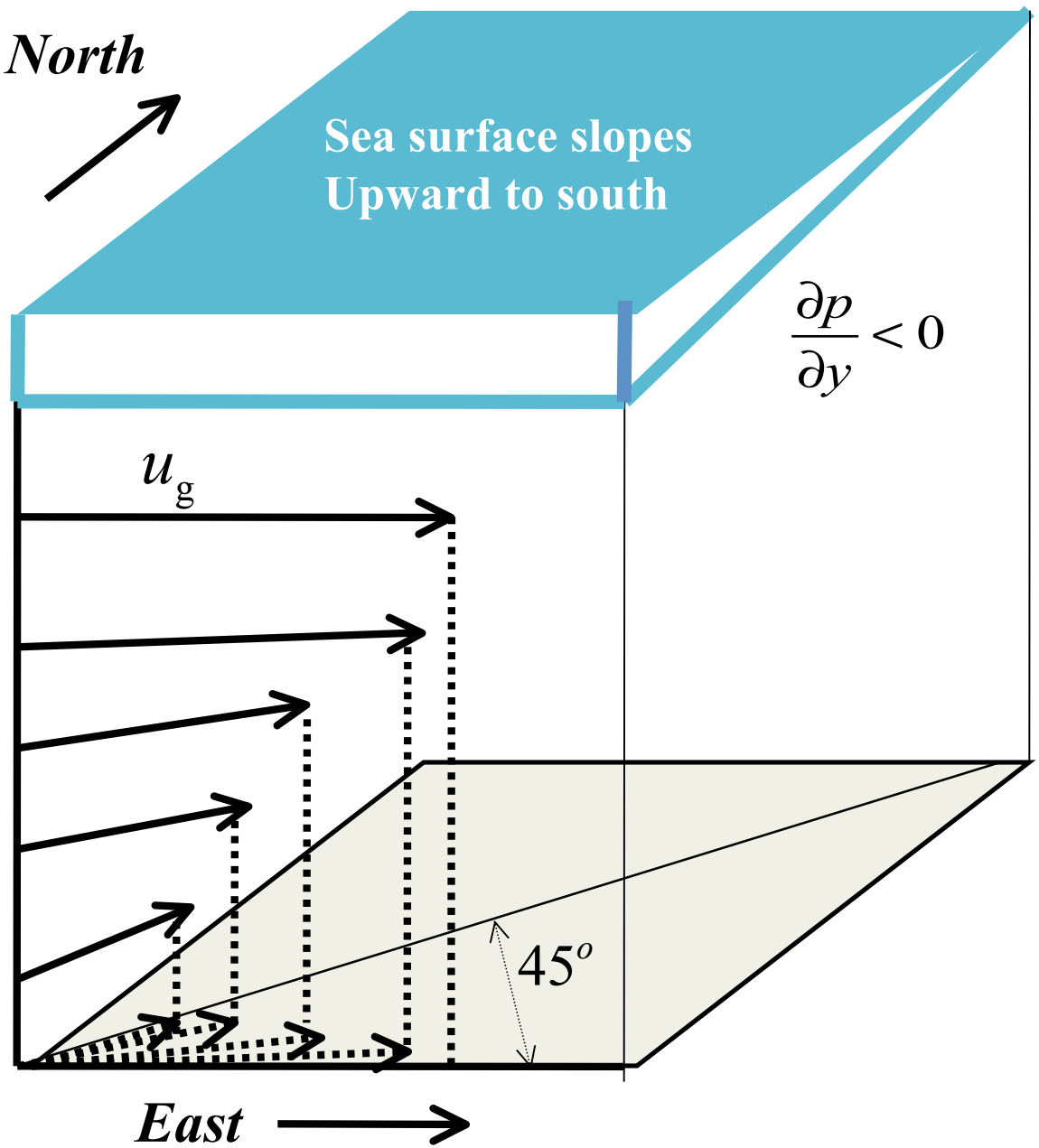


$$\tau_y = \rho_0 k \frac{\partial \mathrm{v}}{\partial z}$$

$$fu = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$



Ekman Dynamics: Bottom Layer



Assumptions

Steady State

Upper Ocean Geostrophic

$$u_g = -\frac{1}{f\rho_0}\frac{\partial p}{\partial y}$$

Deep Ocean

$$-fv = -\frac{1}{\rho}\cancel{\left(\frac{\partial p}{\partial x}\right)} + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

Eddy Viscosity

$$\tau_x = k\frac{\partial u}{\partial z}$$

$$\tau_y = k\frac{\partial v}{\partial z}$$

Equations

$$-f\mathrm{v} = k\frac{\partial^2 u}{\partial z^2}$$

$$fu = fu_g + k\frac{\partial^2 \mathrm{v}}{\partial z^2}$$

Solutions

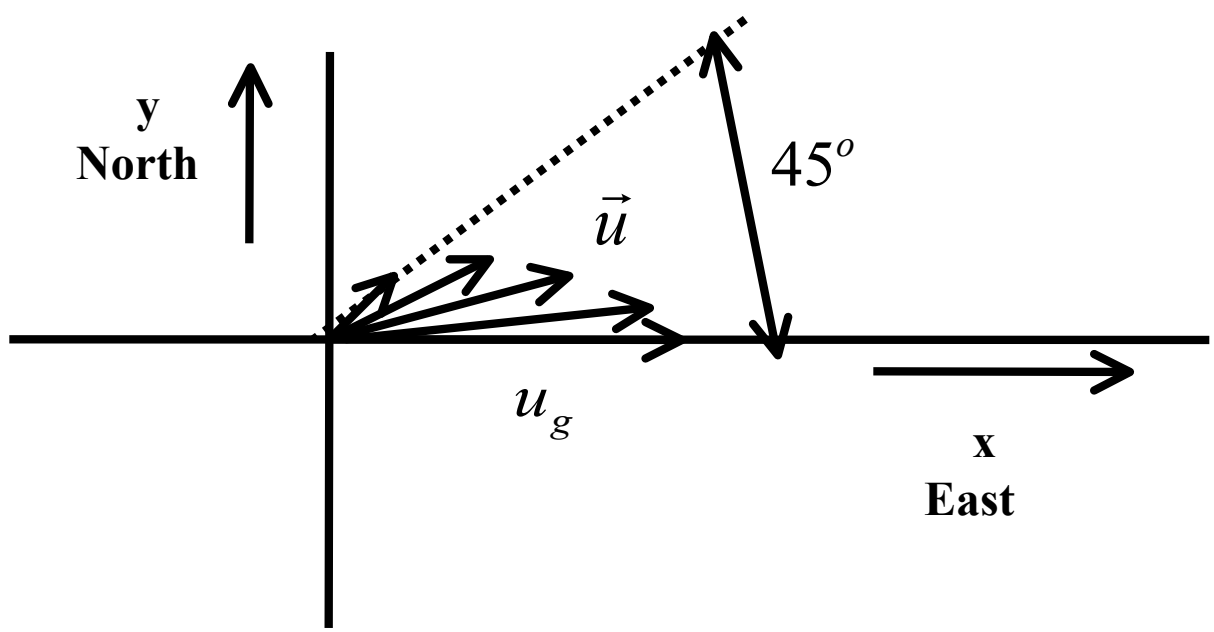
$$\mathrm{u} = u_g[1-\exp(-bz)\cos(bz)]$$

$$\mathrm{v} = u_g \exp(-bz)\sin(bz)$$

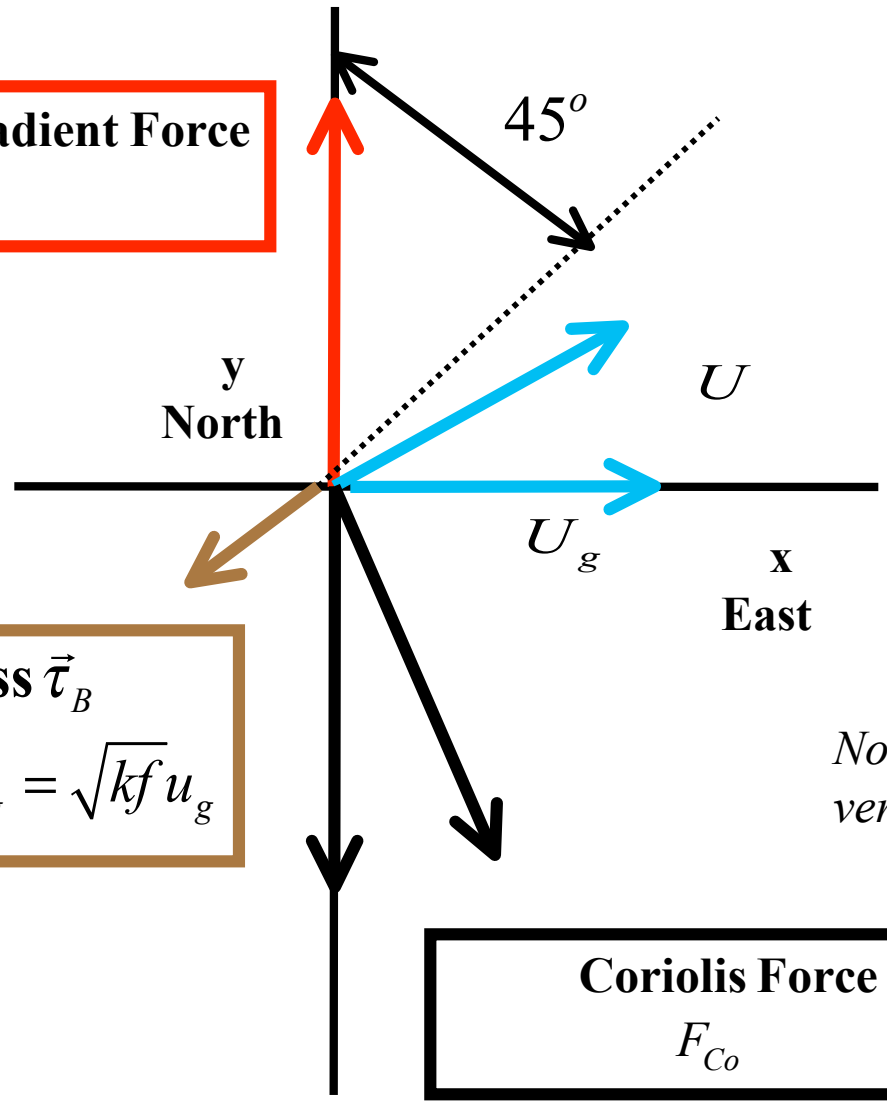
Note z positive upward

$$\mathrm{b}=\sqrt{\frac{\mathrm{f}}{2\mathrm{k}}} \quad \mathrm{bH}>>1$$

View Looking Down



Force Balance



Pressure Gradient Force

F_p

Bottom Stress $\vec{\tau}_B$

$$|\vec{\tau}_B| = \rho_0 k b u_g = \sqrt{kf}\, u_g$$

Coriolis Force

F_{Co}

Geostrophy

$$f u_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$$

Ekman Dynamics

$$-fv = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

Note: Forces are obtained by a vertical integral of above equations

Homework: Along a west-east oriented continental shelf at latitude 42°N, of depth 100 meters, the sea surface has an cross shelf slope of 10^{-5}, increasing to the south. In situ measurements from the T-REMUS vehicle indicate a near bottom eddy viscosity of $k = 10^{-2}$ m^2/sec. Assume that the wind is sufficiently light that it can be neglected. (a) What is the direction and magnitude of the surface current ; (b) what is the direction and magnitude of the bottom current? (c) what is the "effective" thickness of the bottom Ekman layer? (d) calculate the along shelf and cross shelf velocities at z = .5 m, 1m, 10 m off of the bottom.(e) What is the magnitude and direction of the bottom stress? (f) Estimate the magnitude of turbulent velocity at the bottom. (g) What is the direction of the near bottom mass transport anomaly (non geostrophic mass transport)?