Atmospheric Forcing of the Upper Ocean

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MAR 555

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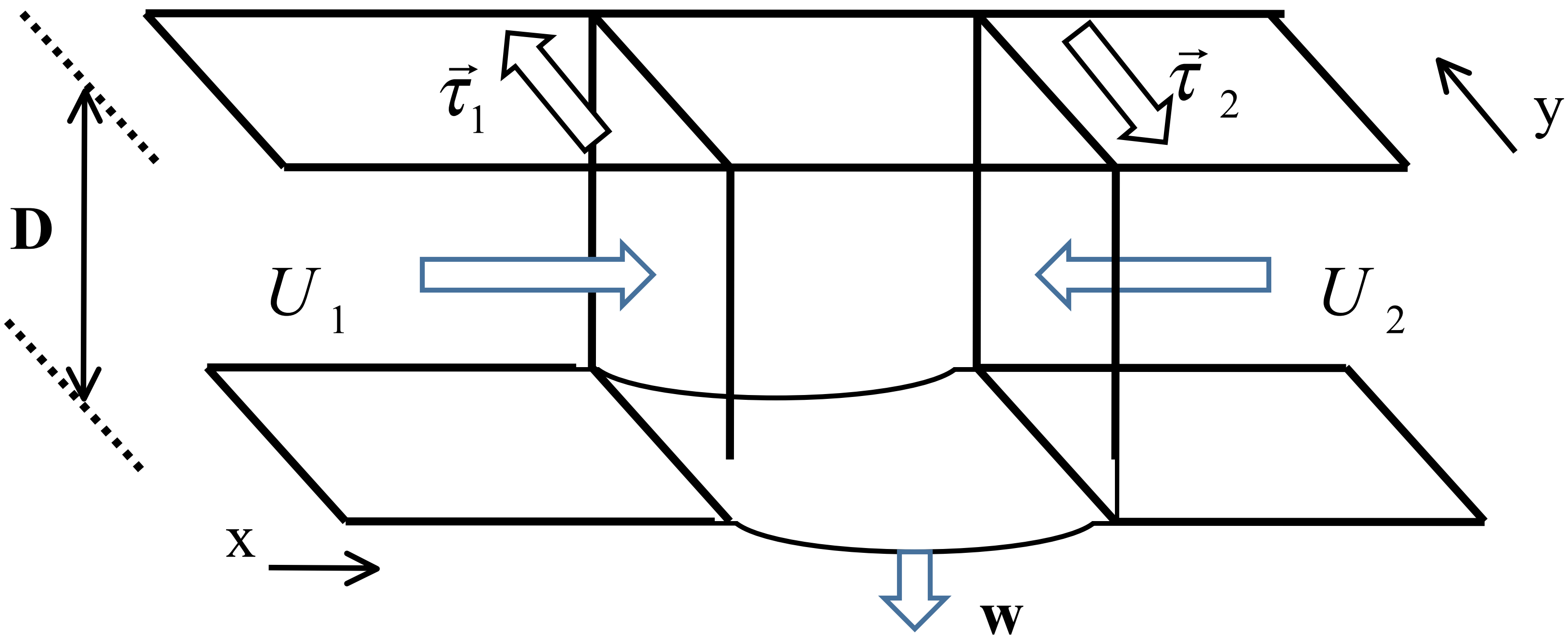
Ekman Dynamics II (IPO-9)

Concept of Ekman Pumping



Vertically Integrated Transport U, V

$$-f\mathrm{v} = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \Rightarrow V = \int \mathrm{v}dz = -\frac{(\tau_x)_{z=0}}{f\rho} = -\frac{\tau_{0x}}{f\rho}$$

$$fu = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \Rightarrow U = \int udz = \frac{(\tau_y)_{z=0}}{f\rho} = \frac{\tau_{0y}}{f\rho}$$

Ekman Pumping

f-plane

$$\frac{\partial}{\partial x}\{\, U = \frac{\tau_{0y}}{f\rho} \,\}$$

$$-$$

$$\frac{\partial}{\partial y}\{\, -V = \frac{\tau_{0x}}{f\rho} \,\}$$

$$\int_{-D}^{0} dz \{\, \frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial \mathrm{y}} + \frac{\partial w}{\partial z} = 0 \,\}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial \mathrm{y}} + \cancel{w(0)} - w(-D) = 0$$

$$w_E = w(-D)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial \mathrm{y}} = w_E = \frac{1}{f\rho}\left(\frac{\partial \tau_{0y}}{\partial x} - \frac{\partial \tau_{0x}}{\partial y}\right)$$

$$\boxed{w_E = \frac{1}{f\rho}(\nabla \times \vec{\tau}_0)_z}$$

$$w_E = \frac{1}{f\rho}(\nabla \times \vec{\tau}_0)_z$$

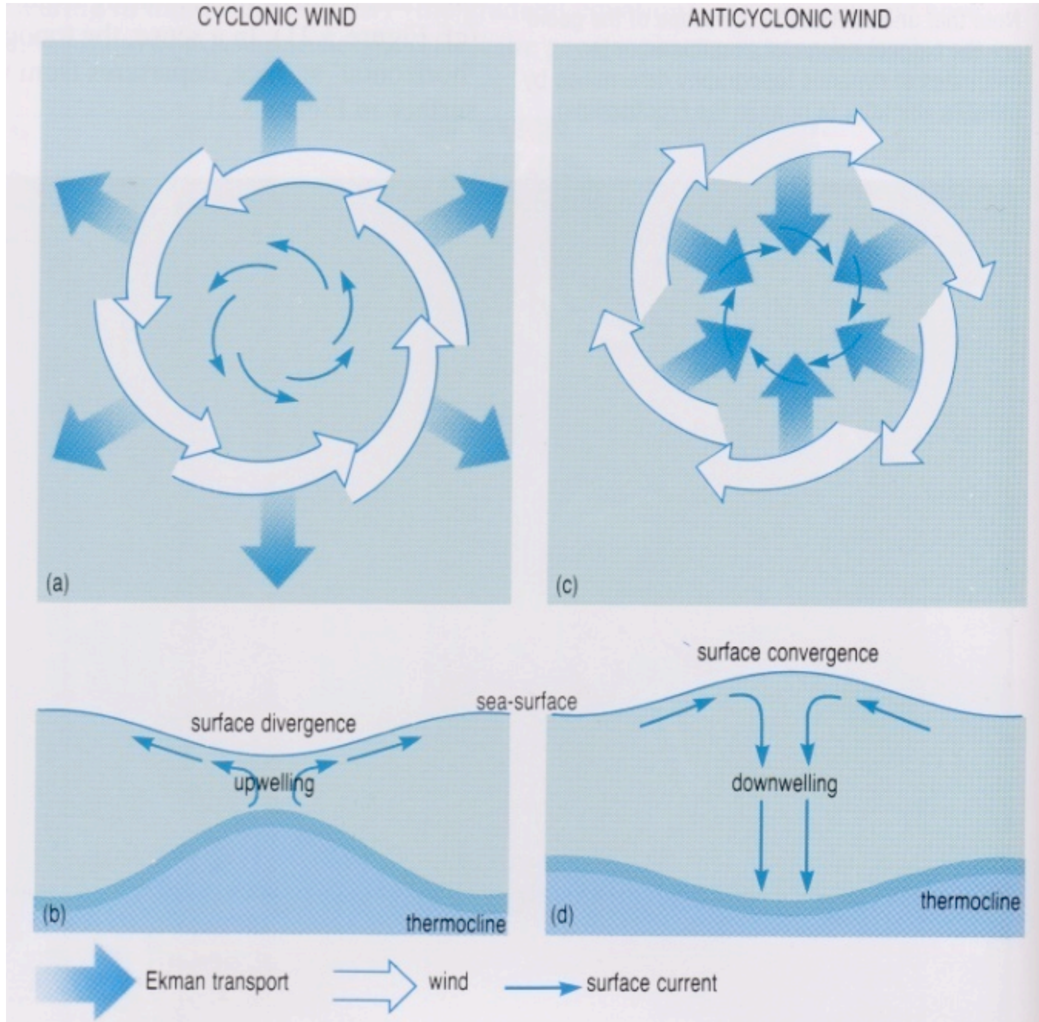
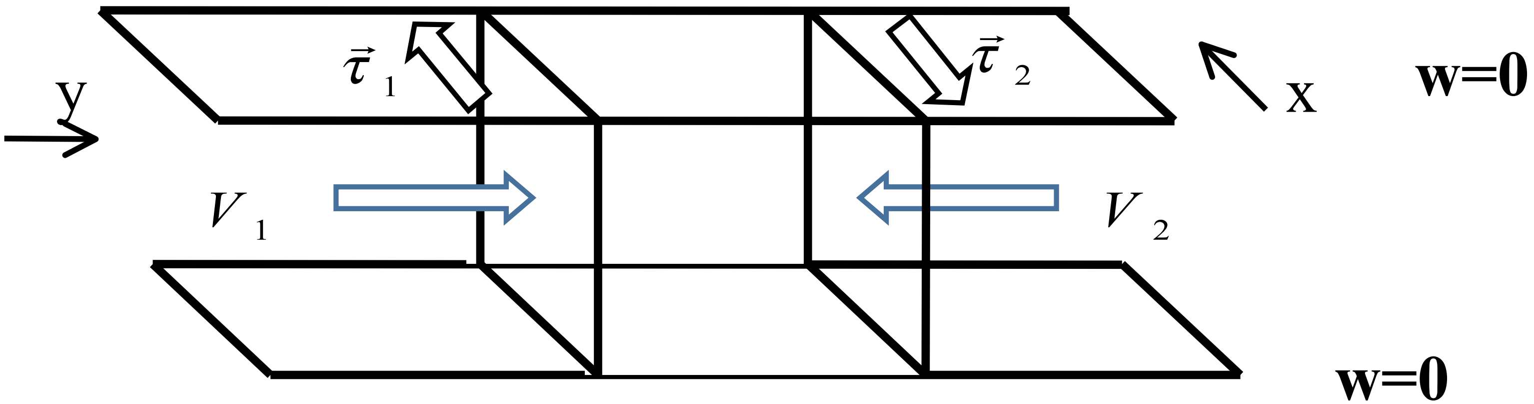


Fig 3.24 Ocean Circulation

Ekman Dynamics

β-plane



Ekman Dynamics *(Sverdrup β-plane Circulation)*

$$\frac{\partial}{\partial x}\{\ fU = \frac{\tau_y}{\rho}\ \}$$

$$-$$

$$\frac{\partial}{\partial y}\{- fV = \frac{\tau_{0x}}{\rho}\ \}$$

$$\int_0^H dz\{\ \frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial \mathrm{y}} + \frac{\partial w}{\partial z} = 0\ \}$$

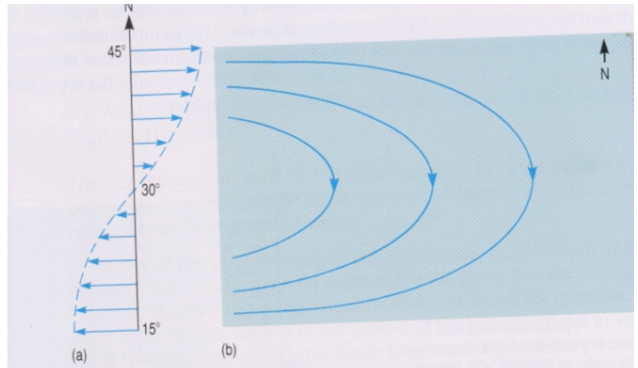
$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial \mathrm{y}} + (\cancel{w_s} - \cancel{w_B}) = 0$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial \mathrm{y}} = 0$$

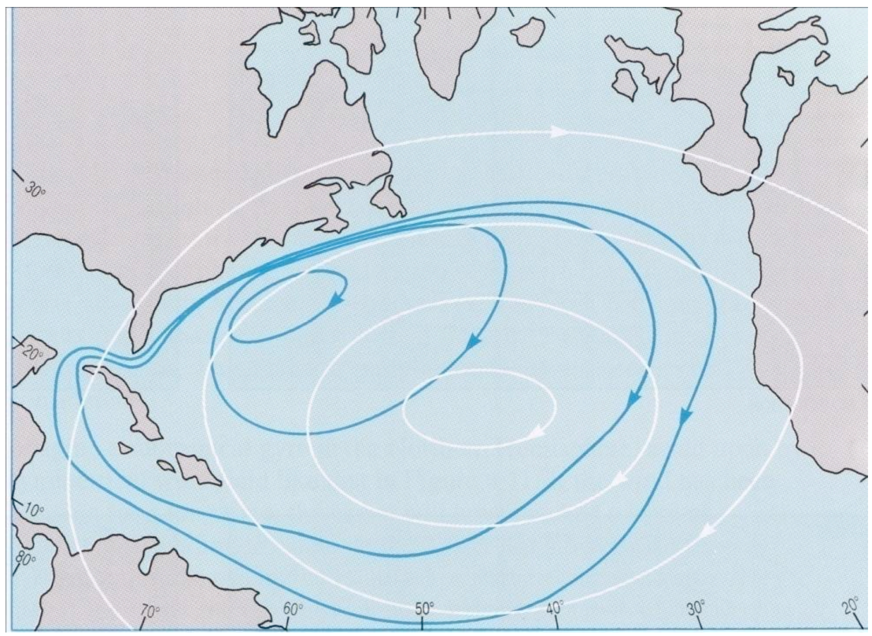
$$f(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial \mathrm{y}}) + V\frac{\partial f}{\partial \mathrm{y}} = \frac{1}{\rho}(\frac{\partial \tau_{0y}}{\partial x} - \frac{\partial \tau_{0x}}{\partial y})$$

$$\boxed{\Rightarrow \beta V = \frac{1}{\rho}(\nabla \times \vec{\tau})_z}$$

where $\beta = \frac{\partial f}{\partial y} = \frac{f}{R}\cot(\theta)$ R = Radius of earth$=6.4 \cdot 10^3 km$



Model of Yearly Average wind Stress over the N. Atlantic



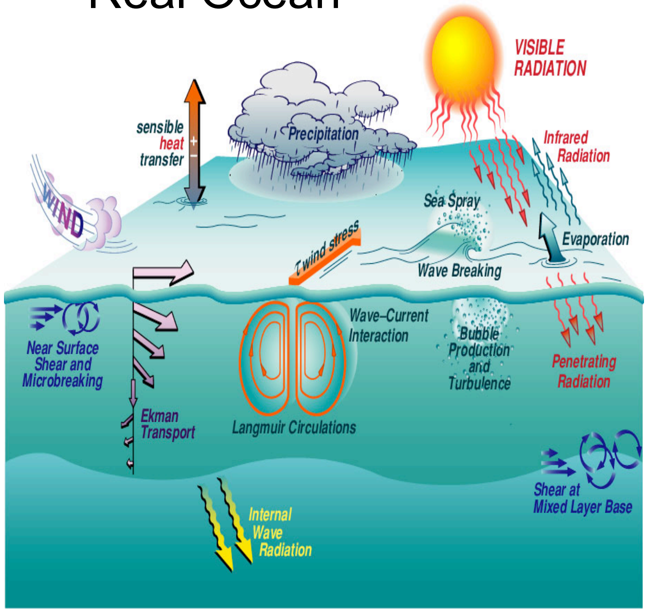
Blue: Schematic of Actual Circulation Gyre; White wind field

**Homework: At Station "A" wind blows to the north with a speed of $U_{10} = 10\frac{m}{sec}$. One degree due east of this location at Station B the wind blows to the north at $U_{10} = 15\frac{m}{sec}$.
What is the Ekman pumping velocity w? Is it upward or downward?**

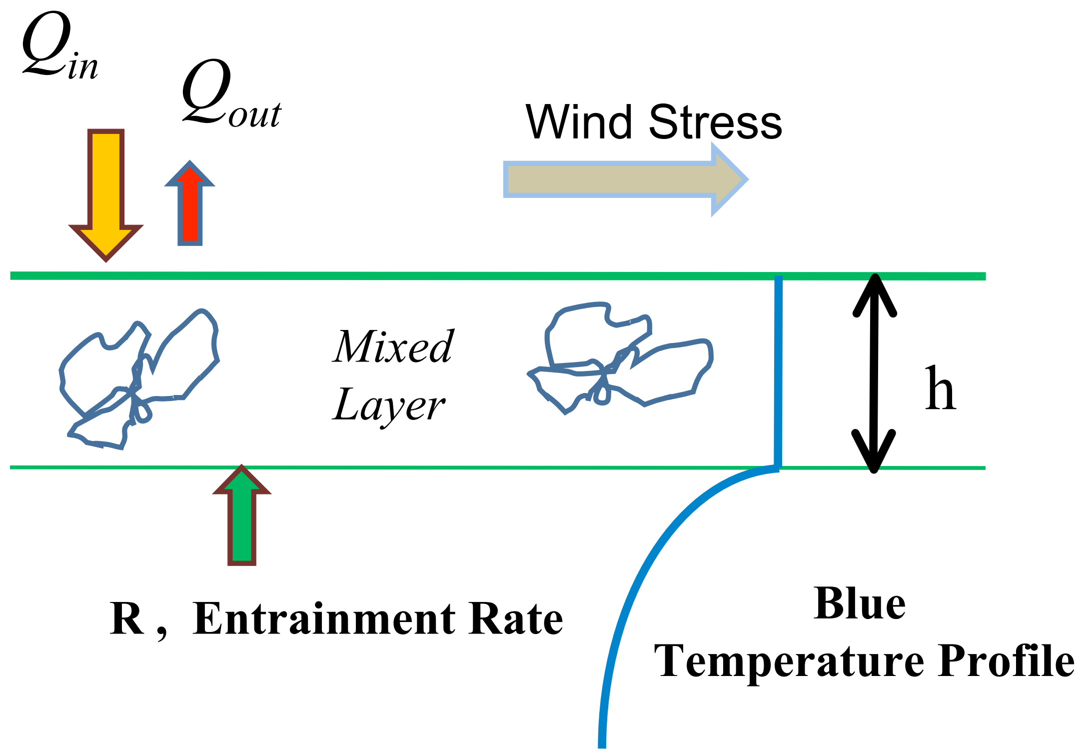
Homework: Suppose for one year the average wind speed over the Atlantic Ocean is eastward and at 45 N $U_{10} = 7\frac{m}{sec}$ while at 30 N $U_{10} = 5\frac{m}{sec}$ (a) What is the magnitude and direction of the vertically integrated transport . (b) If we take the east-west extent of this region of the North Atlantic is 4000 km estimate the volume transport?

Upper Ocean Mixed Layer

“Real Ocean”



“Model” Ocean



Q_{in} $(\frac{W}{m^2})$ Input heat (solar and sensible)

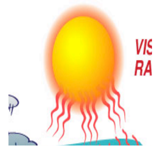
Q_{out} $(\frac{W}{m^2})$ Outgoing heat (long wave and evaporation)

$R = \Delta T \frac{dh}{dt}$ $(\frac{m\,^oC}{\text{sec}})$ Entrainment Rate

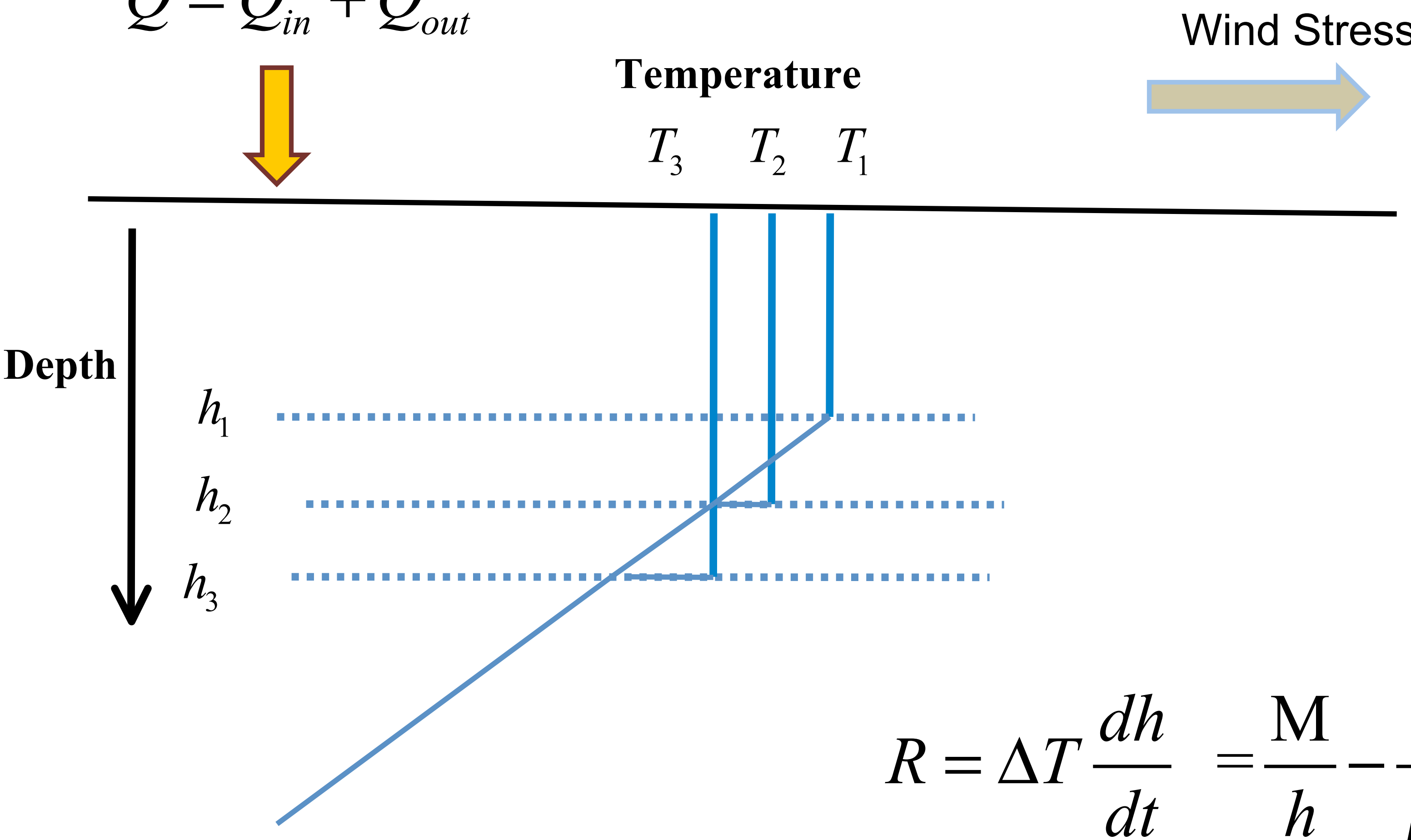
Homework: show that $\frac{Q}{\tilde{n}c}$ has the same units as R.
Note that $c = 4.2 ? 10^3 \frac{J}{kg - {}^oC}$ is the specific heat of seawater and ρ is its density.

Wind Driven Mixed Layer Deepening

(wind forcing dominates heat)



$$Q = Q_{in} + Q_{out}$$

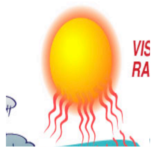


$$R = \Delta T \frac{dh}{dt} = \frac{\mathrm{M}}{h} - \frac{Q}{\rho c}$$

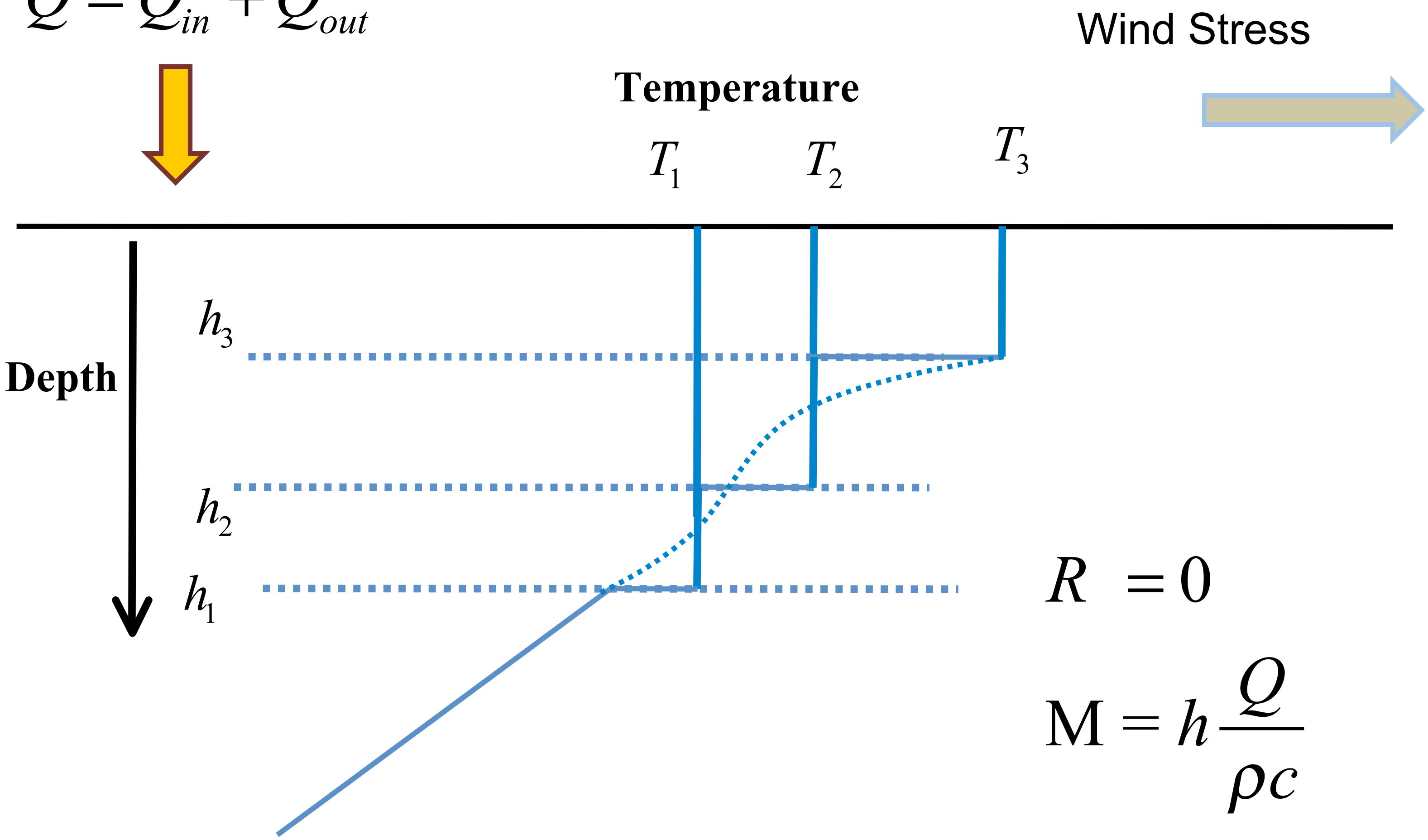
$$M = mixing \;\; at \;\; h$$

Solar Heating, Mixed Layer Shoaling, No entrainment

Heating dominates wind forcing



$$Q = Q_{in} + Q_{out}$$



$$R = 0$$

$$\mathrm{M} = h\frac{Q}{\rho c}$$

Kraus-Turner (1967) Mixed Layer Model

I. Relate temperature flux to buoyancy flux

$$M = 2\int_{-h}^{0} < w'T' > dz$$

$$\boxed{M = \frac{2}{g\alpha}\int_{-h}^{0} B(z)dz}$$

where $B = \frac{g}{\rho} < w'\rho' >$

$= g\alpha < w'T' >$

with $\alpha = \frac{1}{\rho}\frac{\partial \rho}{\partial \mathrm{T}}$

II. Use Flux Richardson number to Relate Buoyancy Flux to Turbulence Production, P

$$R_f = \frac{B}{P} = .25 \qquad \Rightarrow \qquad B = R_f P$$

III. Relate Turbulence Production to Wind Stress Energy input

$$\int_{-h}^{0} P(z)dz = \frac{\tau}{\rho}u^* = (u^*)^3$$

$$\boxed{M = \frac{2}{g\alpha}R_f (u^*)^3}$$

IV. Substitute in Entrainment Equation

$$R = \Delta T \frac{dh}{dt} = \frac{\mathrm{M}}{h} - H$$

$$H = \frac{Q}{\rho c}$$

Homework : show that $\frac{M}{h}$ has the same units as H .

Wind Mixing Dominating
Entrainment R >0

$$R = \Delta T \frac{dh}{dt} \quad = \frac{\mathrm{M}}{h} - H \qquad M = \frac{2}{g\alpha} R_f (u^*)^3$$

$$H = \frac{Q}{\rho c}$$

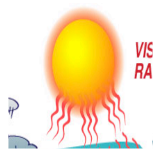
Equilibrium occurs R =0

$$h_e = \frac{M}{H} = \frac{\dfrac{2R_f (u^*)^3}{g\alpha}}{\dfrac{Q}{\rho c}} = \frac{2R_f \rho c \{(\frac{\rho_{air}}{\rho}) C_D (U_{10})^2\}^{\frac{3}{2}}}{g\alpha Q}$$

$$h_e = \frac{2 \cdot .25 * 10^3 \dfrac{kg}{m^3} 4.2 \cdot 10^3 \dfrac{J}{kg \cdot {}^oC} \{(\frac{1}{1000}) 2.5 \cdot 10^{-3}\}^{\frac{3}{2}}}{9.8 \dfrac{m}{\sec^2} 1.5 \cdot 10^{-4} ({}^oC)^{-1}} \frac{(U_{10})^3}{Q}$$

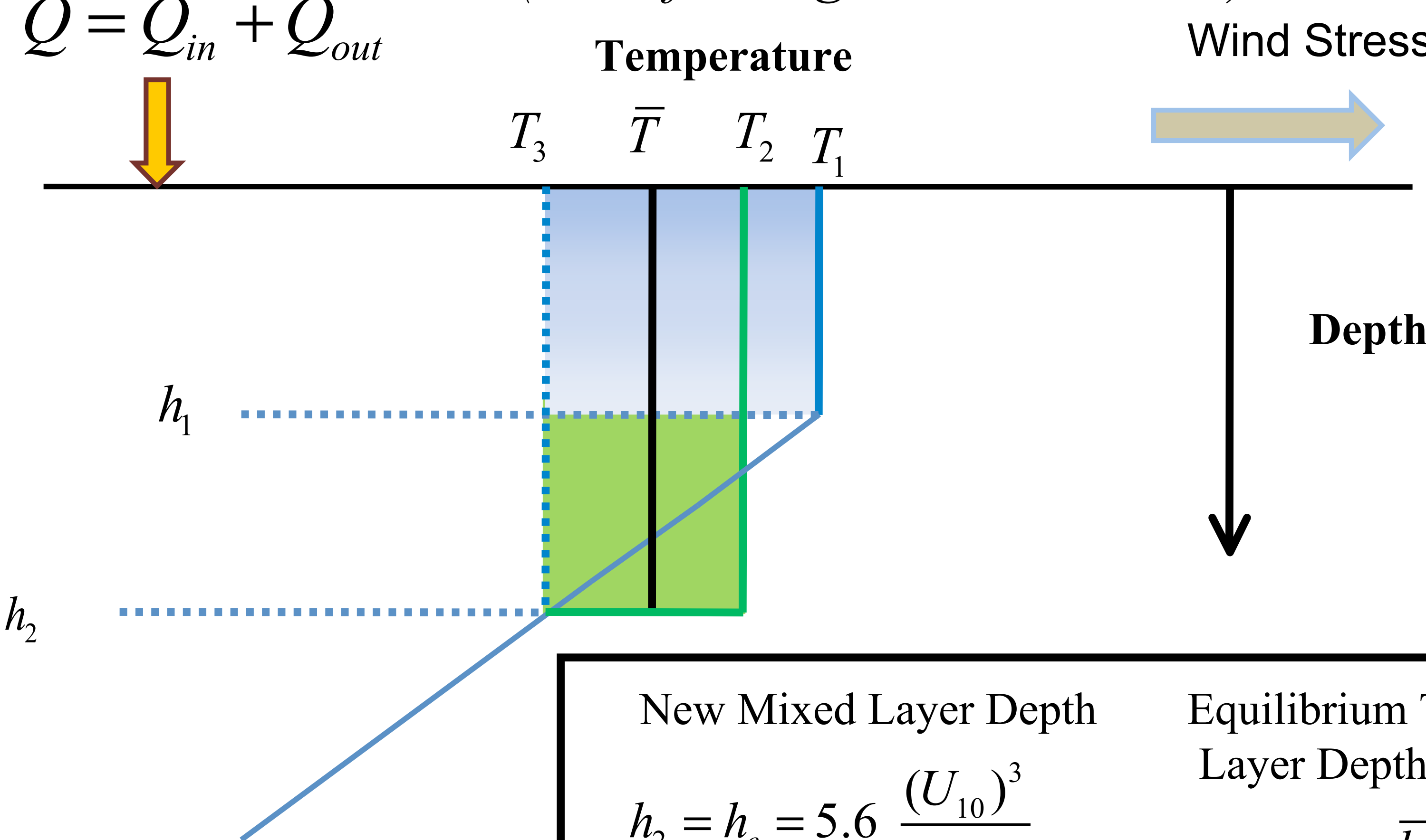
$$\boxed{h_e = 5.6 \frac{(U_{10})^3}{Q}}$$

Homework: Show that the units of the constant 5.6 are $\frac{J \sec^2}{m^4}$



Wind Driven Mixed Layer Deepening

(wind forcing dominates heat)



New Mixed Layer Depth

$$h_2 = h_e = 5.6 \frac{(U_{10})^3}{Q}$$

New Mixed Layer Temperature

$$T_2 = \frac{T_1 h_1 + \bar{T}(\Delta h)}{h_2} \qquad \bar{T} = \frac{1}{2}(T_1 + T_2)$$

Equilibrium Time Period Layer Depth

$$\Delta t = \frac{\bar{h}\Delta T}{H}$$

Time Δt to reach Equilibrium

$$R = \Delta T \frac{dh}{dt} = \frac{\mathrm{M}}{h} - H$$

$$\frac{\Delta h}{\Delta t} \approx \frac{1}{\Delta T}\left(\frac{\mathrm{M}}{\overline{\overline{h}}} - H\right) = \frac{2}{\Delta T}\left(\frac{\mathrm{M}}{h_e}\frac{h_e}{\overline{\overline{h}}} - H\right)$$

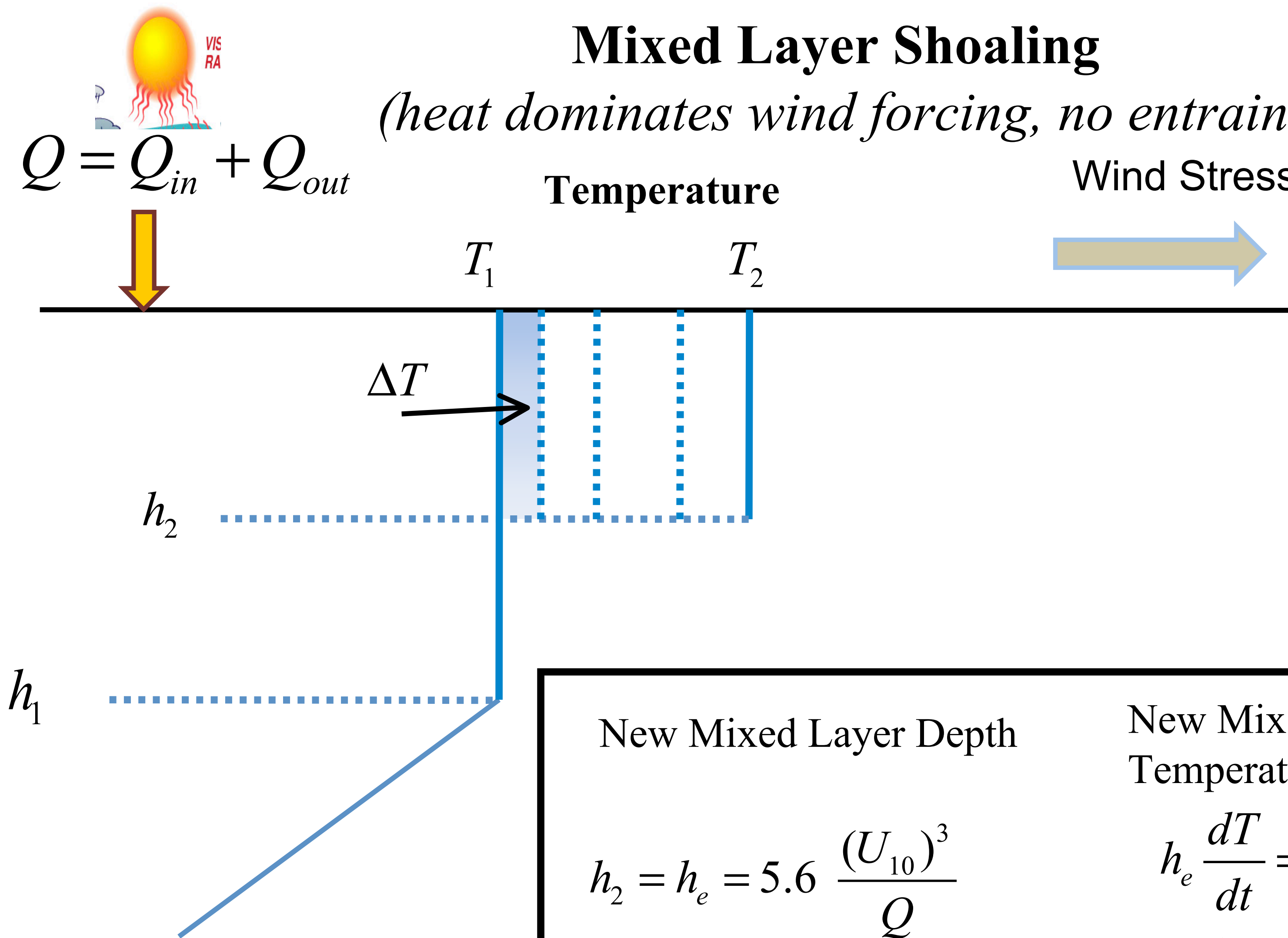
$$= \frac{1}{\Delta T}\left(H\frac{h_e}{\overline{\overline{h}}} - H\right) = \frac{H}{\Delta T}\left(\frac{h_e}{\overline{\overline{h}}} - 1\right)$$

$$= \frac{H}{\Delta T}\left(\frac{h_e - \overline{h}}{\overline{\overline{h}}}\right) = \frac{H}{\Delta T}\left(\frac{\Delta h}{2\overline{\overline{h}}}\right) \Rightarrow$$

$$\boxed{\Delta t = \frac{\overline{h}\,\Delta T}{H} \qquad \overline{h} = \frac{1}{2}(h_1 + h_2)}$$

Mixed Layer Shoaling

(heat dominates wind forcing, no entrainment)



New Mixed Layer Depth

$$h_2 = h_e = 5.6\,\frac{(U_{10})^3}{Q}$$

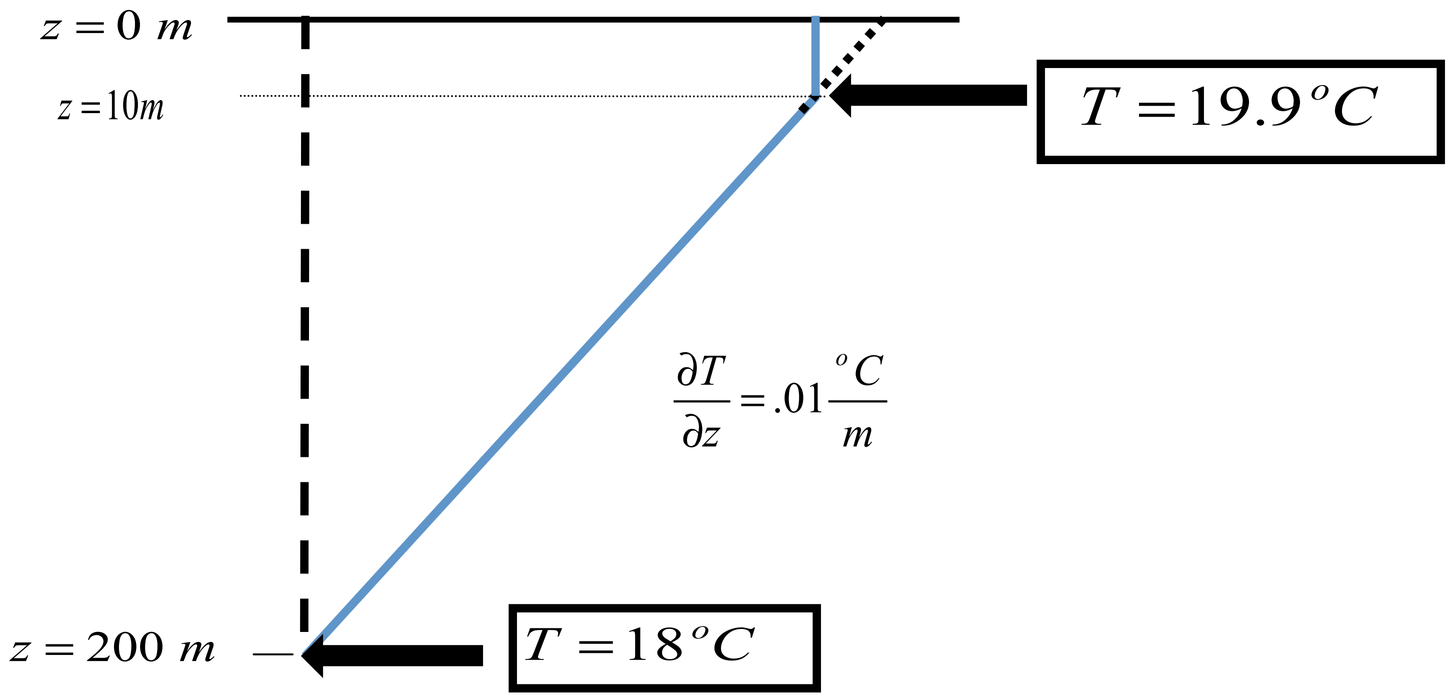
New Mixed Layer Temperature

$$h_e \frac{dT}{dt} = H$$

$$h_e \frac{\Delta T}{\Delta t} = H$$

$$\Delta T = \frac{H \Delta t}{h_e}$$

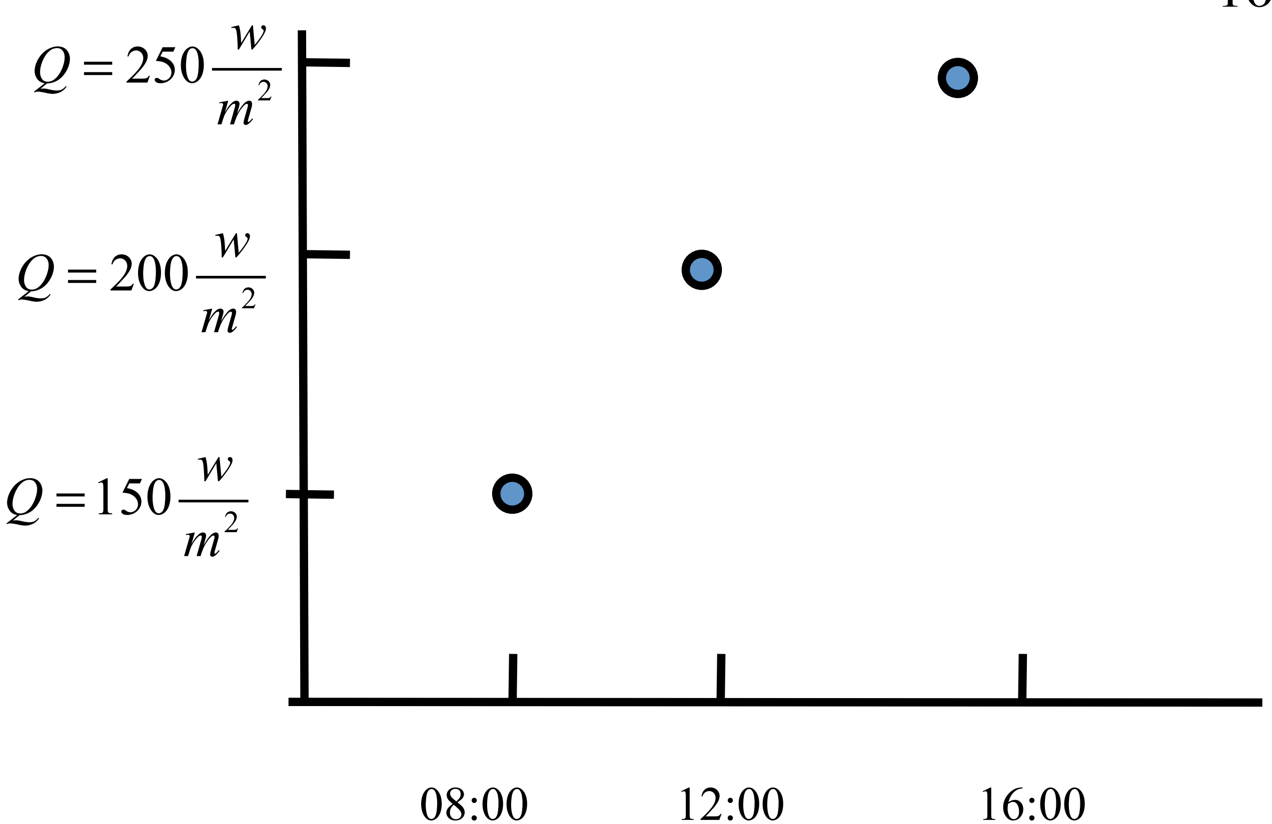
Homework. A 08:00 the ocean has temperature profile shown below with a mixed layer depth initially at z =10 m, a mixed layer temperature of $T = 19.9^oC$ and gradient below of $\frac{\partial T}{\partial z} = .01\frac{^oC}{m}$. There are two daytime forecasts for heating and wind speeds, which are shown in the following two slides. Calculate and plot versus time the mixed layer depth and temperature for each forecast .



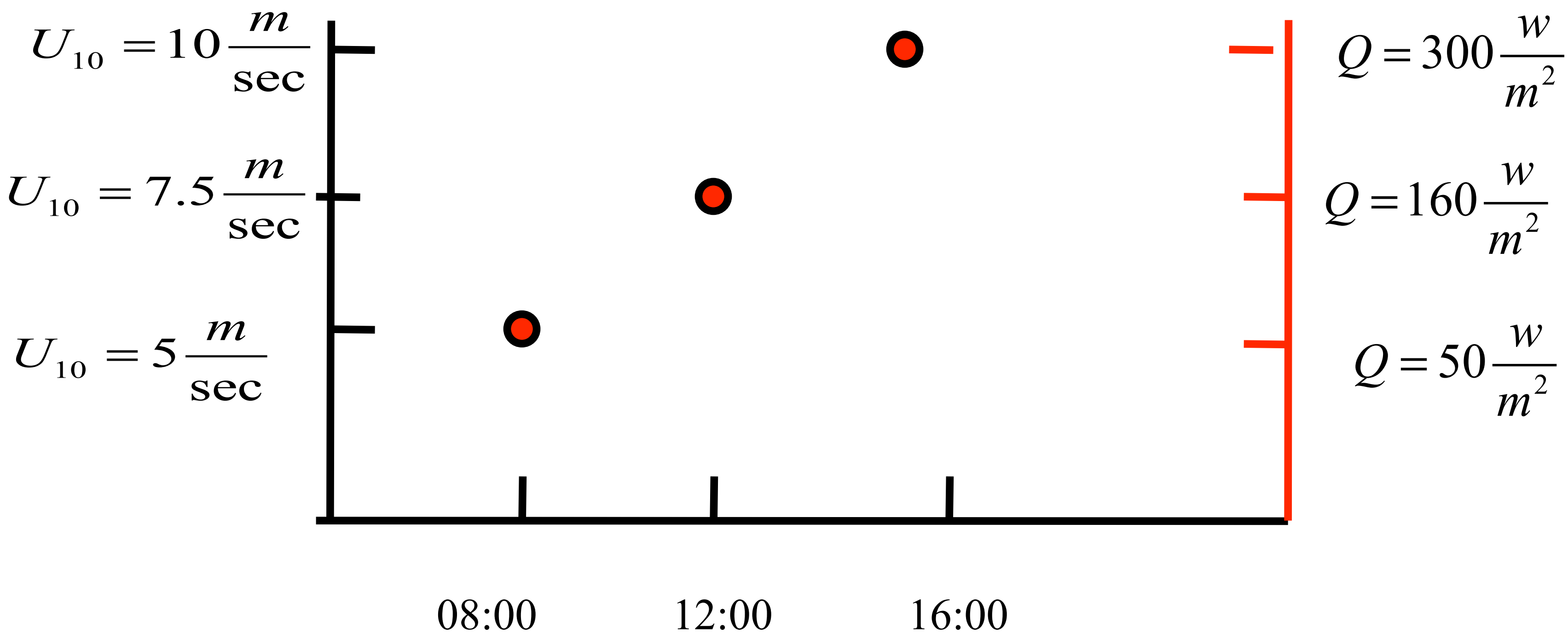
Forecast I

Wind Speed Constant at

$$U_{10} = 7\,\frac{m}{\sec}$$



Forecast II



$U_{10} = 10 \frac{m}{\sec}$
$U_{10} = 7.5 \frac{m}{\sec}$
$U_{10} = 5 \frac{m}{\sec}$
$Q = 300 \frac{w}{m^2}$
$Q = 160 \frac{w}{m^2}$
$Q = 50 \frac{w}{m^2}$
08:00
12:00
16:00