Math and Physics Review

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General Physical Oceanography
MAR 555

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A Brief Math and Physics Review

First: Some Mathematics

• Cartesian Coordinates
• 3 D Notation
• Tensors, Vectors, Scalars
• Polar
• Spherical
• Meaning of Derivative
• Lagrangian versus Eulerian
• The total Derivative
• Meaning of Integral

Second: Some Physics

• Displacement, velocity, acceleration
• Force and momentum, angular momentum
• Energy: kinetic and Potential
3 D Cartesian Coordinate System

What is a vector? Give an example.

What is scalar? Give an example.

What is a tensor? Give an example.

Note: we typically use x, y, z as spatial coordinates, t as time.
Spherical Coordinates

\[ \vec{x} = (x, y, z) = (r, \phi, \phi) \]
Some Useful Formulae

**Circle**
- Circumference = $2\pi r$
- Area = $\pi r^2$

**Sphere**
- Area = $4\pi r^2$
- Volume = $\frac{4}{3} \pi r^3$
Angle Units
$2\pi$ radians = 360 degrees

$\beta$- plane

$f = 2\Omega \sin(\theta) + \beta y$

$2\Omega = \frac{2 \times 2\pi}{24 \text{ hours}} = \frac{2 \times 2\pi}{24 \times 3600 \text{ seconds}} = 1.5 \times 10^{-4} \text{ rad / sec}$
Figure 3.8  Plan view showing inertial motion observed in the Baltic Sea at about 57° N. The diagram shows the path of a parcel of water; if this path was representative of the general flow, the surface water in the region was both rotating and moving north-north-west. The observations were made between 17 and 24 August, 1933, and the tick marks on the path indicate intervals of 12 hours.
What is a first derivative \( \frac{dz}{dx} \)?

Example: Slope of a surface wave

\[ z \text{ (vertical up)} \]

\[ x \text{ (East)} \]

The derivative is a limit.

\[
\frac{dz}{dx} = \text{limit of} \quad \frac{\delta z}{\delta x} \quad \text{as} \quad x_2 \to x_1 \quad \text{and} \quad z_2 \to z_1
\]

\[ \delta z = z_2 - z_1 \]

\[ \delta x = x_2 - x_1 \]
Partial Derivative:
This is a derivative with respect to one variable when there is more than one independent variable.
Eulerian: Measurement at a fixed point

Lagrangian: Follow the fluid parcel and measure $T$.

The Lagrangian derivative

$$\frac{dT}{dt} = \lim_{(t_2 \to t_1)} \frac{T_2 - T_1}{t_2 - t_1}$$
The Total Derivative
1 D Example

\[ u = \frac{.5}{\text{sec}} \frac{m}{\text{sec}} \]

\[ \begin{array}{c|c}
\text{Cold water} & \text{Warm water} \\
T = 10^\circ C & T = 20^\circ C \\
x = 2m & x = 7m
\end{array} \]

What is the Lagrangian derivative \( \frac{dT}{dt} \)?

Answer: \( \frac{dT}{dt} = \frac{(20 - 10)^\circ C}{10 \text{ seconds}} = 1^\circ C \text{ sec}^{-1} \)

What is the Eulerian derivative \( \frac{\partial T}{\partial t} \) at A? at B?
The Lagrangian derivative $\frac{dT}{dt}$ in 3D.

\[
\frac{dT}{dt} = \frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \\
= \frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T
\]

Summation on “i” is implied when index is repeated.

Note for 1D:  \[
\frac{dT}{dt} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x}
\]

If $\frac{dT}{dt} = 0$ & $u = constant$

$T = T(x - ut)$
Problem: Suppose you measure the temperature of the water at some fixed point and you observe your thermometer changing. You then decide to float with the water current, and find that your thermometer does not change in its reading of temperature. Using the concept of the Lagrangian derivative what can you say about $\frac{dT}{dt}$? A clever SMAST student says “you know if you know what the current $u$ is you can actually use that measurement at a fixed point to be able to measure its spatial variability in $x$.“ Explain.
What is an integral?

Answer the area under the curve.

\[ W = \int_{x_1}^{x_2} f(x) \, dx = \text{Area Under curve} \]

One can also have two and three dimensional integrals.
Now some physics

Velocity = first derivative of position slope on $x$ versus $t$ plot

$$v = \frac{dx}{dt}$$

Acceleration = first derivative of velocity

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

= second derivative of distance

$$\frac{d^2x}{dt^2}$$
What is a force?

Your weight is a force but your mass is not!
Units: of Newtons MKS

**Newton’s Second Law**

\[
\vec{F} = m \vec{a}
\]

Note that force is a vector. It depends on magnitude and direction.

What is energy?

When a force is applied over a distance parallel it produces energy.

Energy can be neither created or destroyed but changed from one form to another.

**Types:** kinetic, potential, heat
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Concept of Spectra (Frequency or Wavenumber)
Spectra shows sinusoidal components of variance
\[ y = A \sin(\omega t) \]

\[ \omega = \frac{2\pi}{T} \quad T = \text{Period} \]

\[ y = A \sin(kx) \]

\[ k = \frac{2\pi}{\lambda} \quad \lambda = \text{wavelength} \]

\[ A = ? \]

\[ \lambda = ? \]

\[ k = ? \]
Fourier Analysis

\[ = \]

\[ + \]

\[ + \]

\[ + \]

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\[ + \]