Math and Physics Review

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MAR 555

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A Brief Math and Physics Review

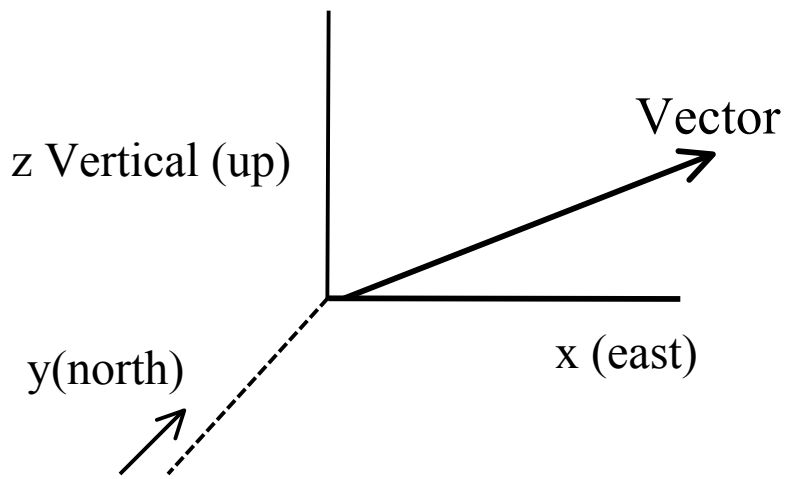
First: Some Mathematics

•Cartesian Coordinates
•3 D Notation
•Tensors, Vectors, Scalars
•Polar
•Spherical
•Meaning of Derivative
• Lagrangian versus Eulerian
•The total Derivative
•Meaning of Integral

Second: Some Physics

•Displacement, velocity, acceleration
•Force and momentum, angular momentum
•Energy: kinetic and Potential

3 D Cartesian Coordinate System



3 D Notation

$\vec{u} = (u_x, u_y, u_z) = (u_1, u_2, u_3)$

sometimes written as u_i

$i = 1, 2, 3$

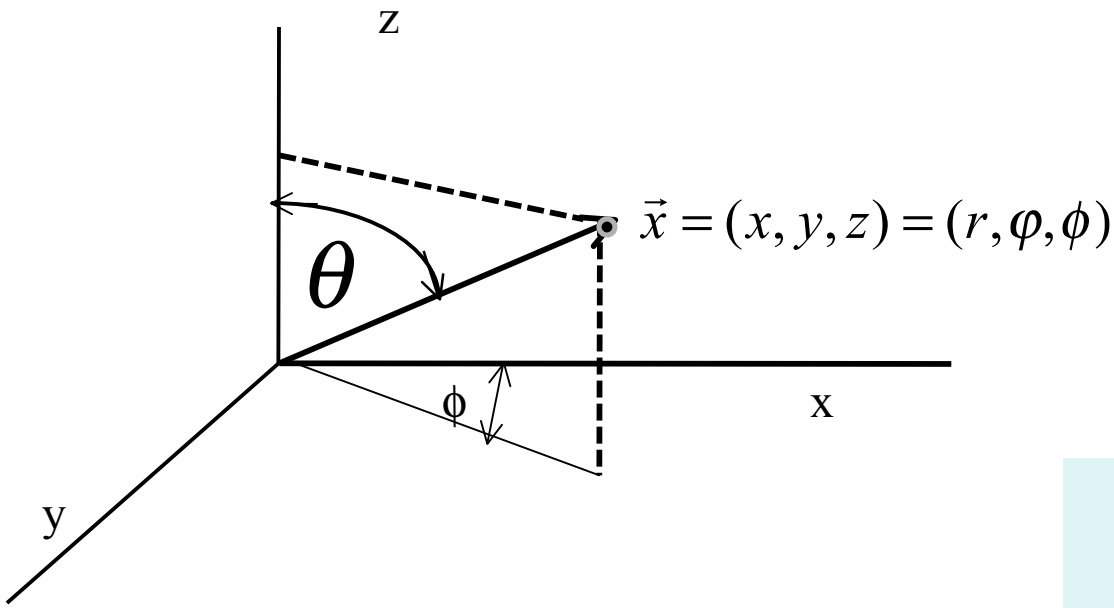
What is a vector? Give an example.

What is scalar? Give an example.

What is a tensor? Give an example.

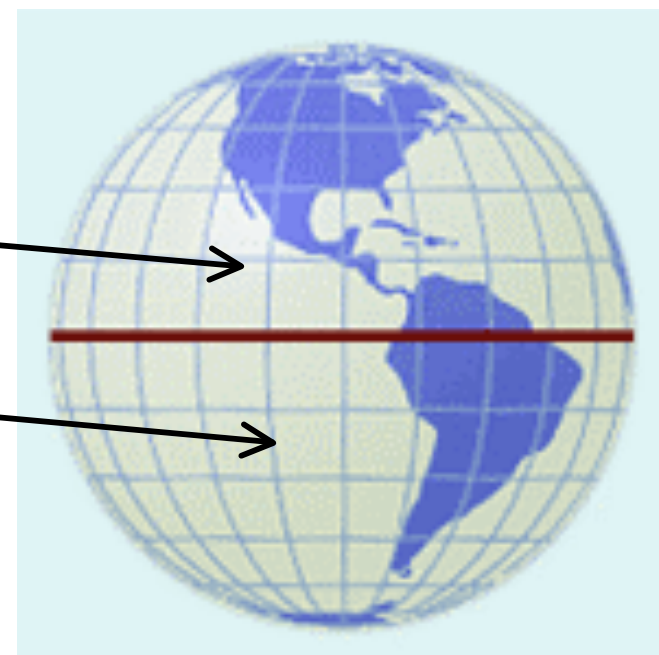
Note: we typically use x, y, z as spatial coordinates, t as time.

Spherical Coordinates

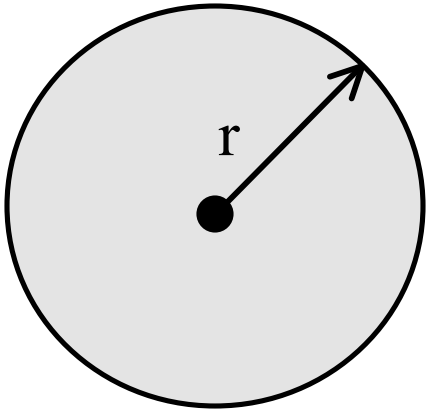


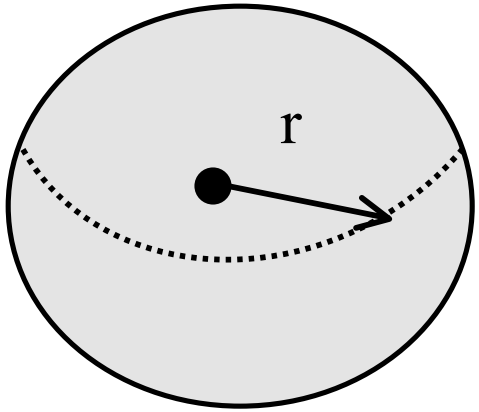
θ Latitude

ϕ Longitude



Some Useful Formulae





Circle

$$Circumference = 2\pi r$$

$$Area = \pi r^2$$

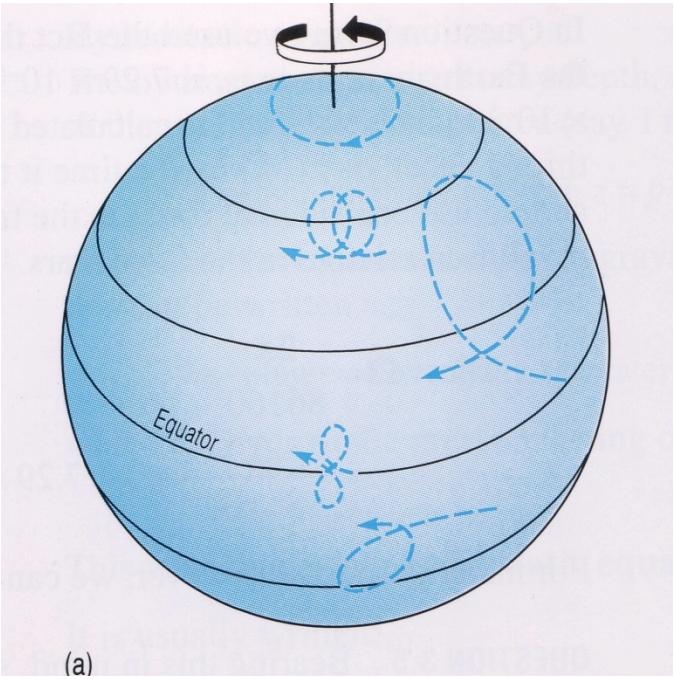
Sphere

$$Area = 4\pi r^2$$

$$Volume = \frac{4}{3}\pi r^3$$

Angle Units
2π radians = 360 degrees

$$f = 2\Omega$$



f- plane

$$f = 2\Omega \sin(\theta)$$

β- plane

$$f = 2\Omega \sin(\theta) + \beta y$$

$$2\Omega = \frac{2*2\pi}{24 hours} = \frac{2*2\pi}{24*3600 \sec onds} = 1.5 \cdot 10^{-4} rad / \sec$$

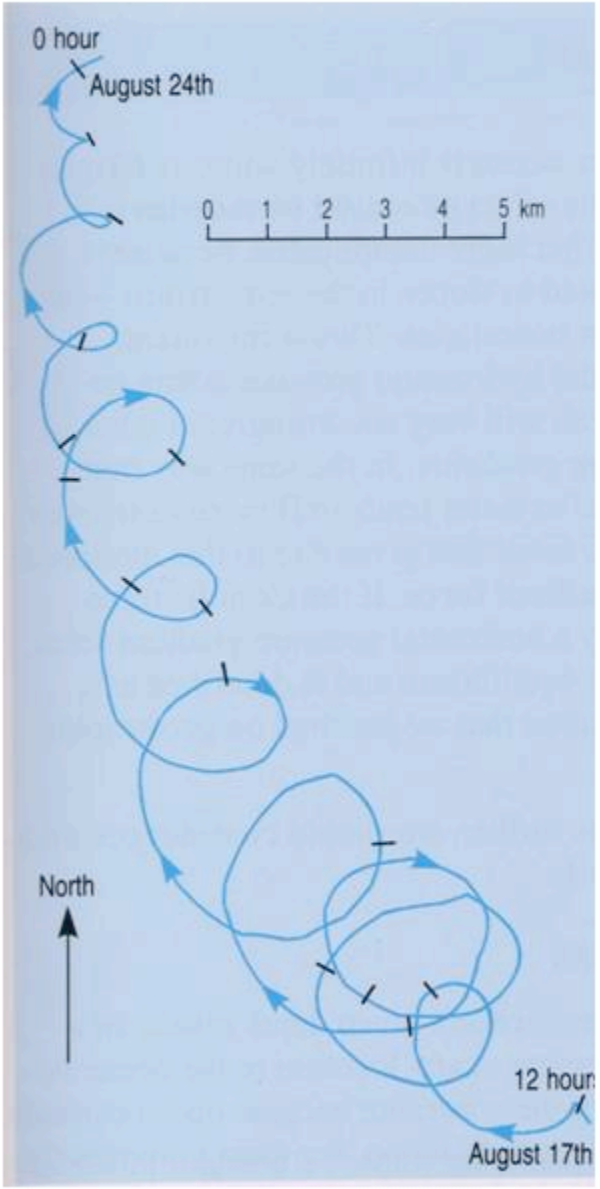
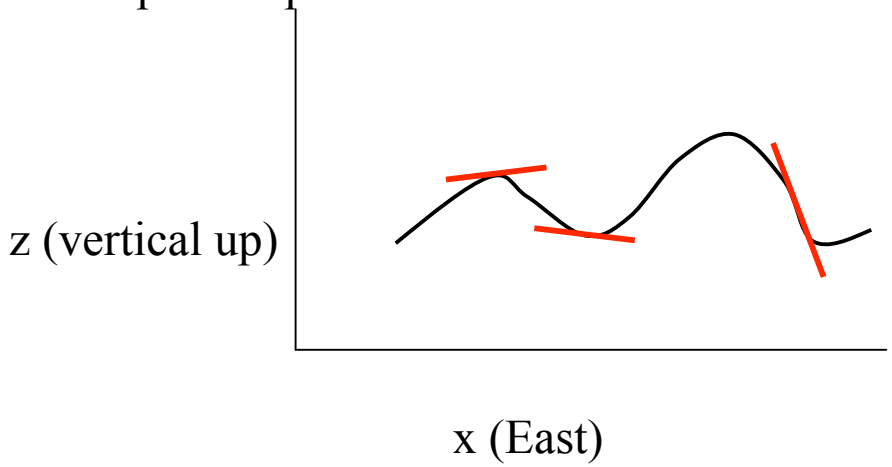


Figure 3.8 Plan view showing inertial motion observed in the Baltic Sea at about 57° N. The diagram shows the path of a parcel of water; if this path was representative of the general flow, the surface water in the region was both rotating and moving north-north-west. The observations were made between 17 and 24 August, 1933, and the tick marks on the path indicate intervals of 12 hours.

From "Ocean Circulation"

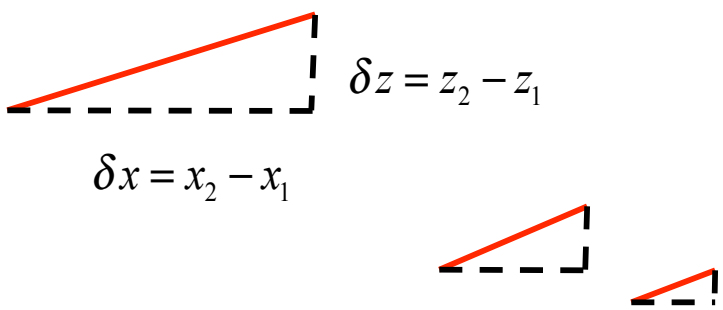
What is a first derivative $\frac{dz}{dx}$?

Example: Slope of a surface wave



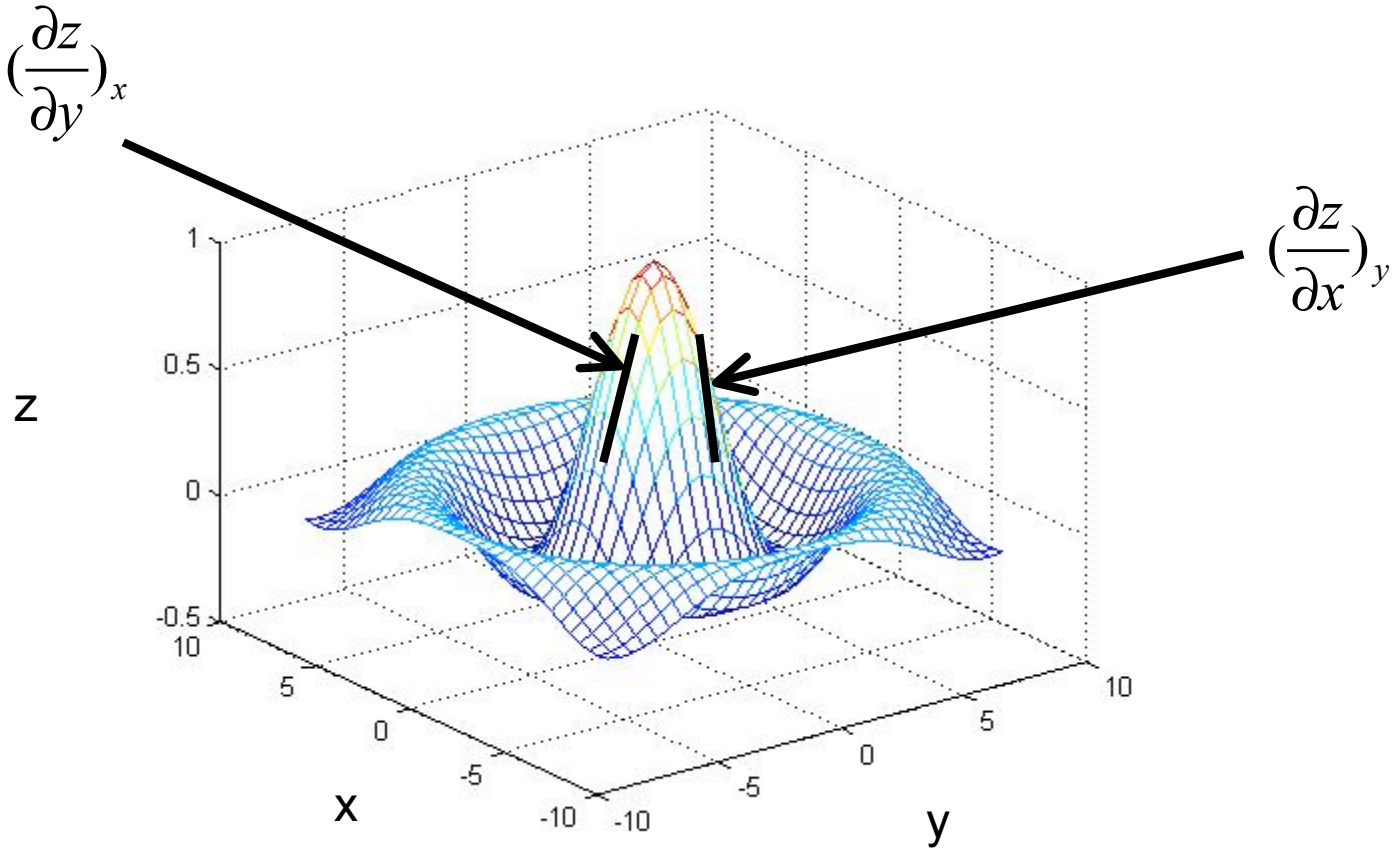
The derivative is a limit.

$$\frac{dz}{dx} = \text{limit of } \frac{\delta z}{\delta x} \quad \text{as } x_2 \to x_1 \; and \; z_2 \to z_1$$

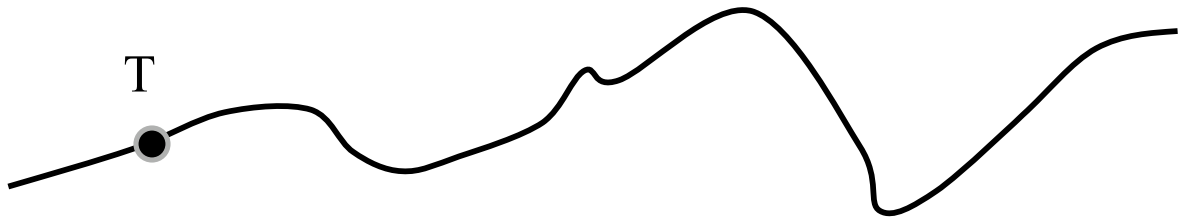


Partial Derivative:

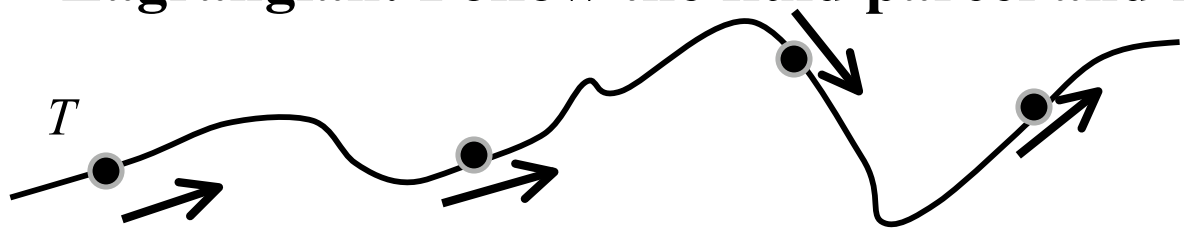
This is a derivative with respect to one variable when there is more than independent variable



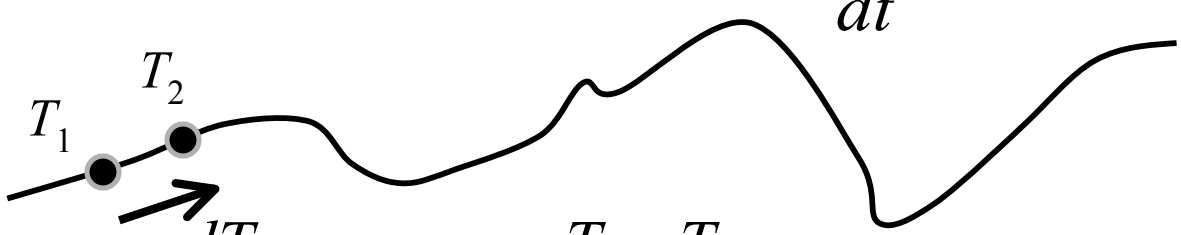
Eulerian: Measurement at a fixed point



Lagrangian: Follow the fluid parcel and measure *T*.

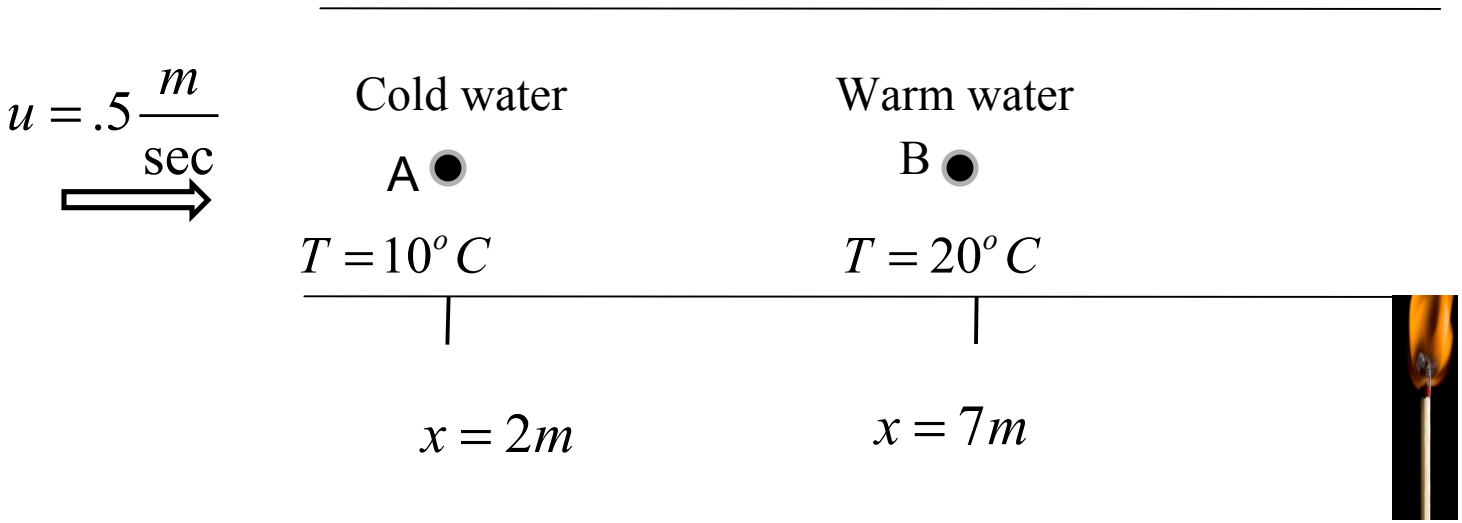


The Lagrangian derivative $\frac{dT}{dt}$



$$\frac{dT}{dt} = \lim_{(t_2 \to t_1)} \frac{T_2 - T_1}{t_2 - t_1}$$

The Total Derivative
1 D Example

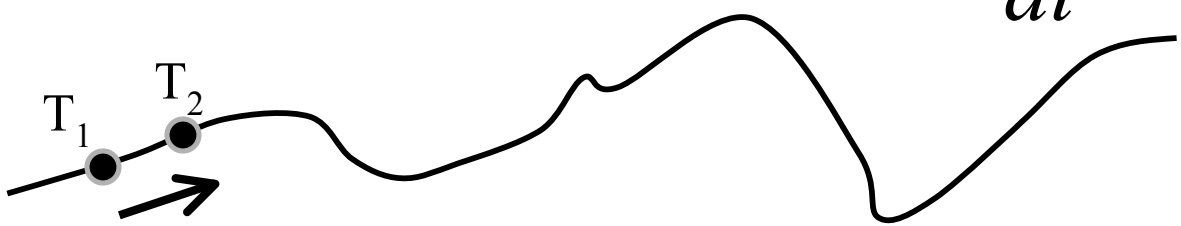


What is the Lagrangian derivative $\frac{dT}{dt}$?

$$Answer: \quad \frac{dT}{dt} = \frac{(20-10)^o C}{10 \text{ seconds}} = 1 \frac{^o C}{\text{sec}}$$

What is the Eulerian derivative $(\frac{\partial T}{\partial t})_{A,B}$ at A? at B?

The Lagrangian derivative $\frac{dT}{dt}$ in 3D.



$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_x \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z}$$

$$= \frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} \quad = \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T$$

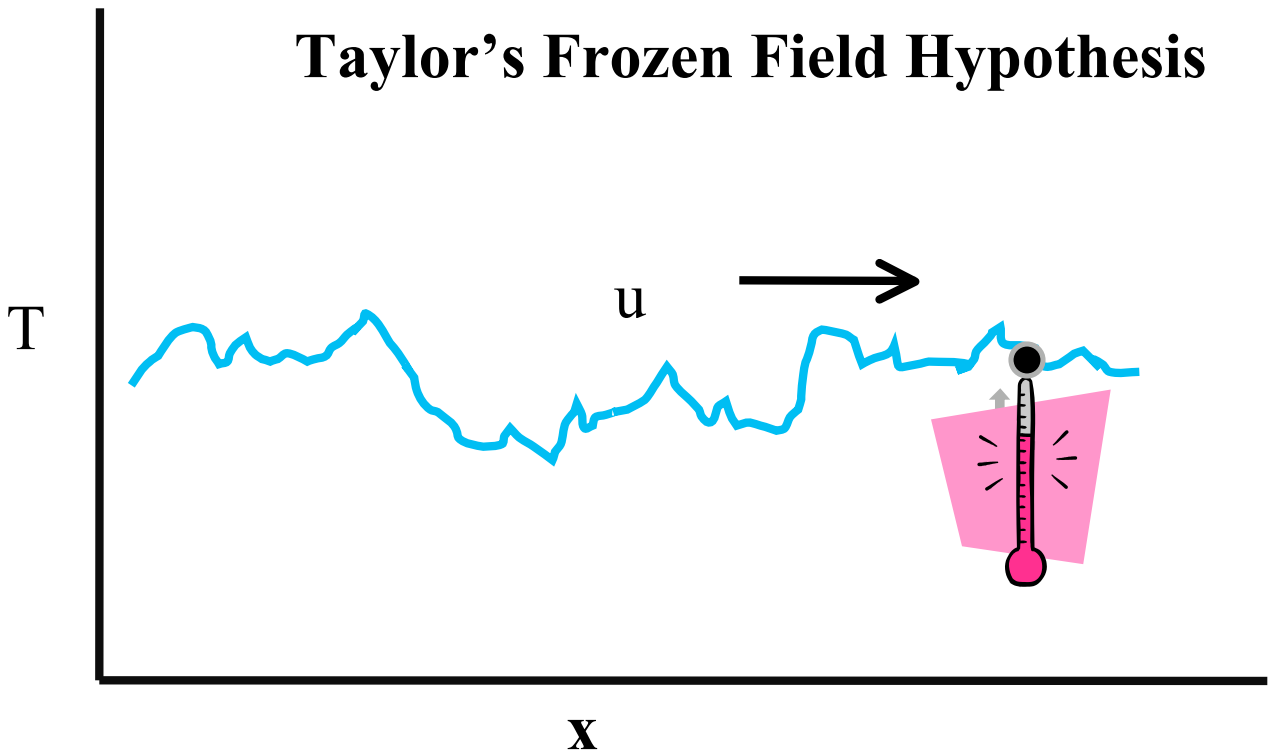
Summation on "i" is implied when index is repeated

Note for 1D: $\frac{dT}{dt} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x}$

If $\frac{dT}{dt} = 0$ ***& u = constant***

T = T(x - ut)

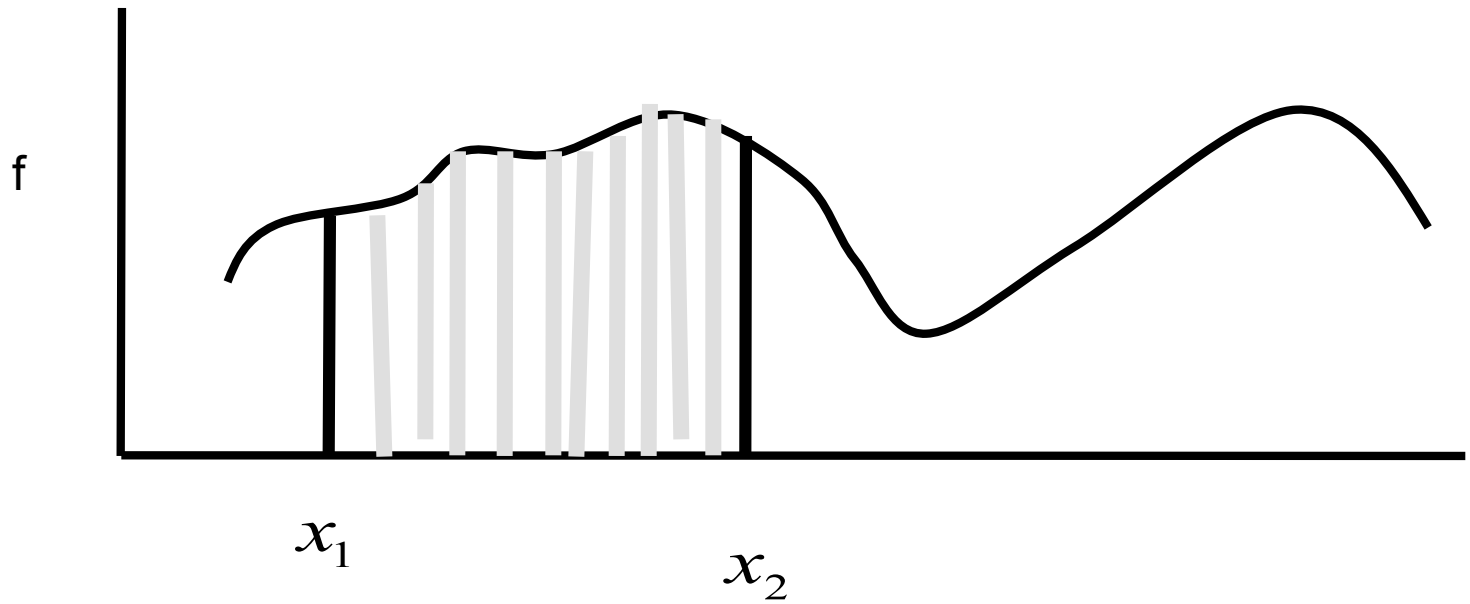
Taylor's Frozen Field Hypothesis



Problem: Suppose you measure the temperature of the water at some fixed point and you observe your thermometer changing. You then decide to float with the water current, and find that your thermometer does not change in its reading of temperature. Using the concept of the Lagrangian derivative what can you say about $\frac{dT}{dt}$? A clever SMAST student says "you know if you know what the current u is you can actually use that measurement at a fixed point to be able to measure its spatial variability in x." Explain.

What is an integral?

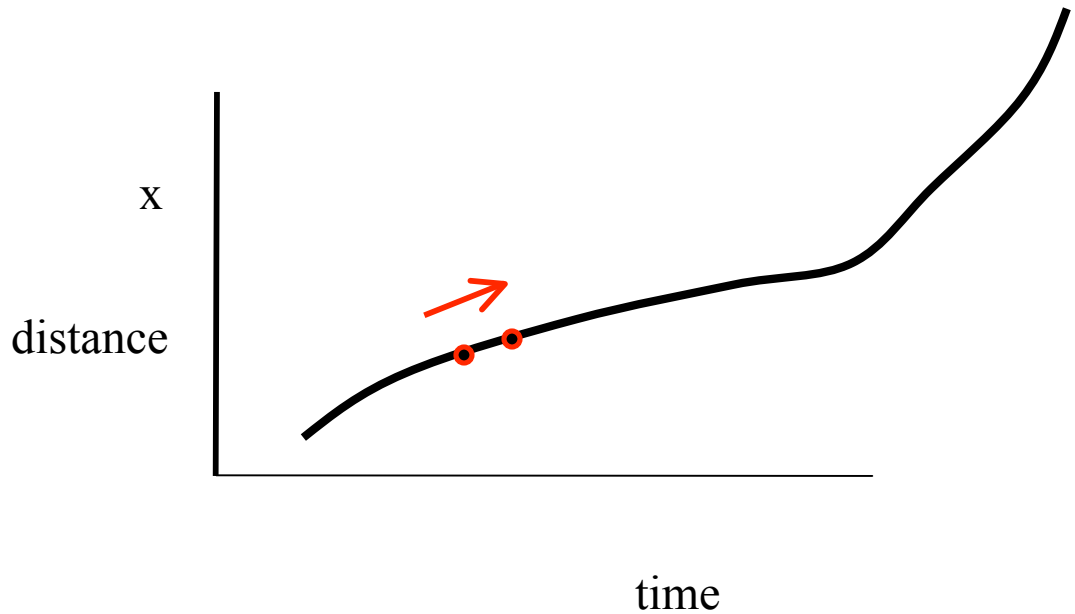
Answer the area under the curve .



$$W = \int_{x_1}^{x_2} f(x)dx = \text{ Area Under curve}$$

One can also have two and three dimensional integrals.

Now some physics



$$v = \frac{dx}{dt}$$

Velocity= first derivative of position slope on x versus t plot

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Acceleration = first derivative of velocity
= second derivative of distant

What is a force?

Your weight is a force but your mass is not!
Units: of Newtons MKS

Newton's Second Law

$$\text{Force} = (\text{mass}) \text{ x } (\text{accelration})$$

$$\vec{F} = m \quad \vec{a}$$

Note that force is a vector. It depends on magnitude and direction.

What is energy?

When a force is applied over a distance parallel it produces energy.

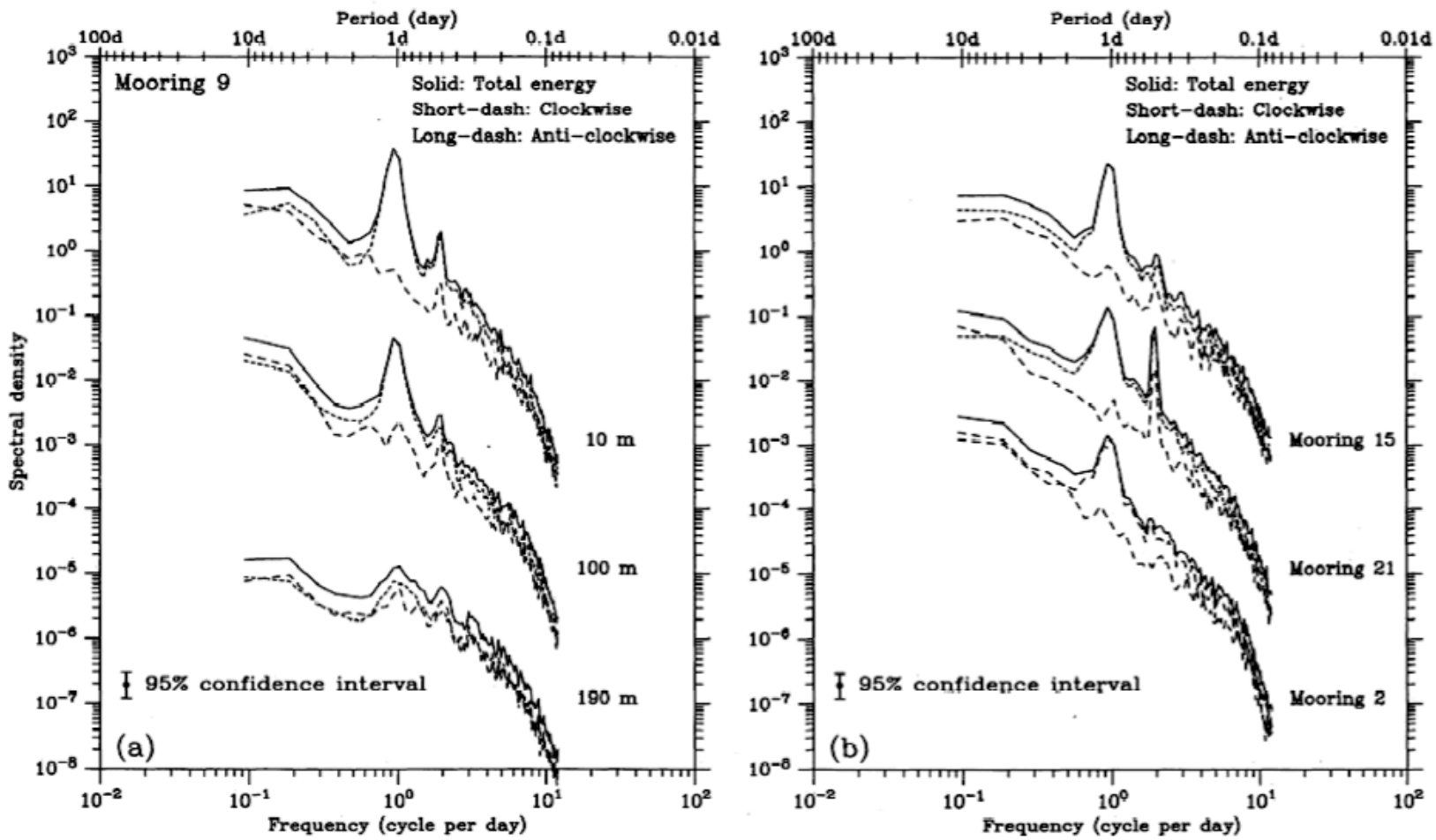
Energy can be neither created or destroyed but changed from one form to another.

Types: *kinetic ,potential , heat*

Dominant Ocean Forces	
Gravity	Gives rise to pressure gradients, buoyancy, and tides.
Coriolis	Results from motion in a rotating coordinate system.
Friction	Is due to relative motion between two fluid parcels. Wind stress is an important frictional force.
Other Forces	
Atmospheric Pressure	Results in inverted barometer effect.

Conservation Laws In Physical Oceanography		
Conservation of Mass :	Leads to Continuity Equation.	
Conservation of Energy:	Conservation of heat leads Heat Budgets.	
	Conservation of mechanical energy leads to Wave Equation	
Conservation of Momentum: (mass times velocity)	Leads to Momentum (Navier-Stokes) Equation.	
Conservation of Angular Momentum (spin): Angular velocity times angular mass	Leads to Conservation of Vorticity.	

Concept of Spectra (Frequency or Wavenumber)
Spectra shows sinusoidal components of variance



$$y = A\sin(\omega t)$$

$$\omega = \frac{2\pi}{\mathrm{T}}$$ T =Period

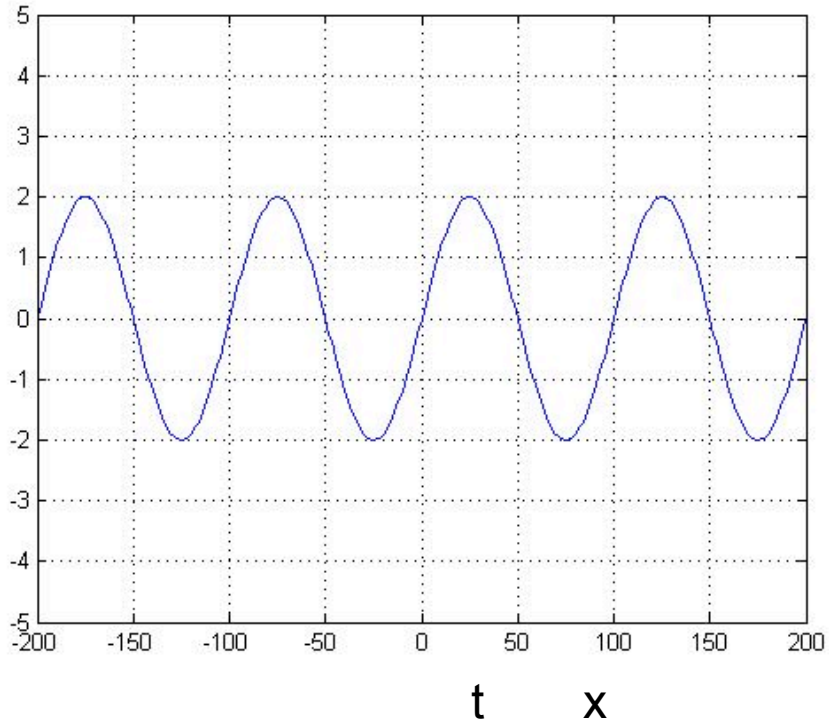
$$y = A\sin(kx)$$

$$k = \frac{2\pi}{\lambda}$$ λ =wavelength

$A = ?$

$T = ?$

$\omega = ?$



$A = ?$

$\lambda = ?$

$k = ?$

Fourier Analysis