Review: Lecs 19-23

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MAR 555

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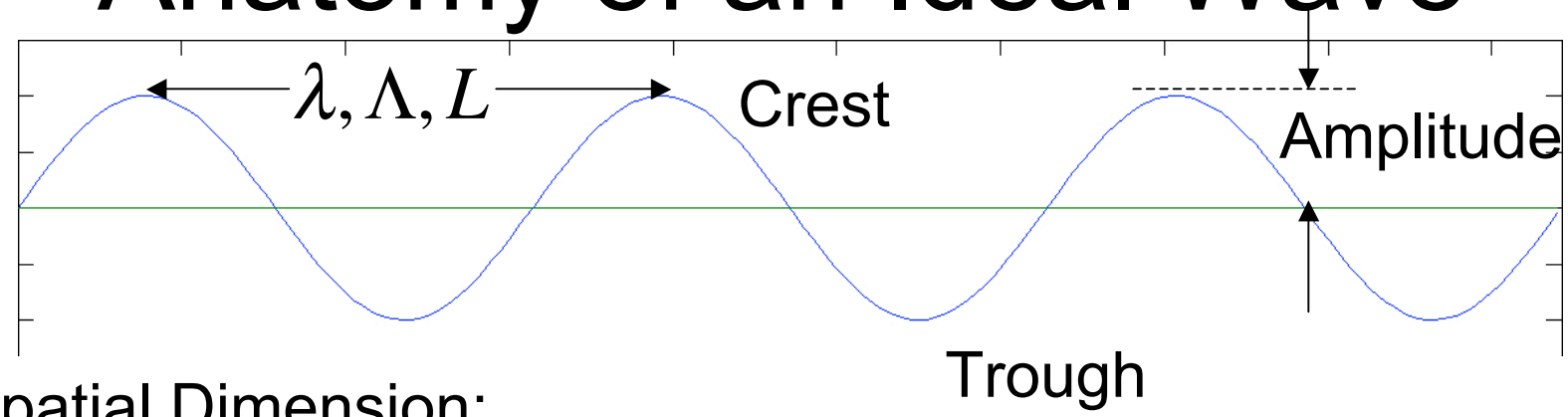
Anatomy of an Ideal Wave



Spatial Dimension:

λ, Λ, L (Wavelength) [m]

$k = \dfrac{2\pi}{\Lambda}$ (Wavenumber) [1/m]

Temporal Dimension:

T (Period - time between successive crests measured by fixed observer) [s]

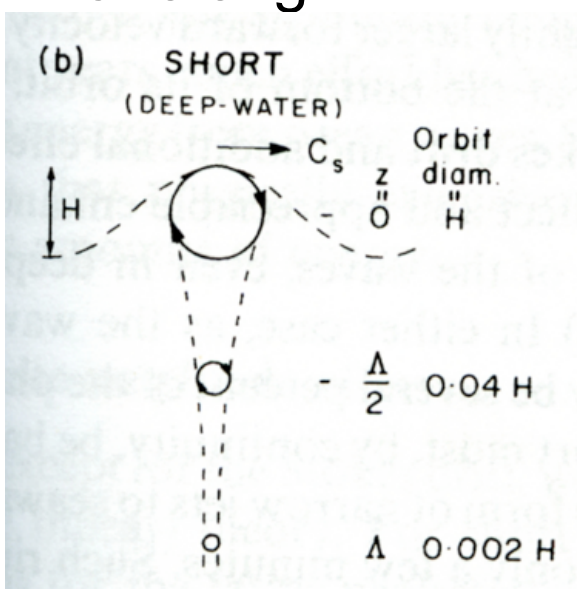
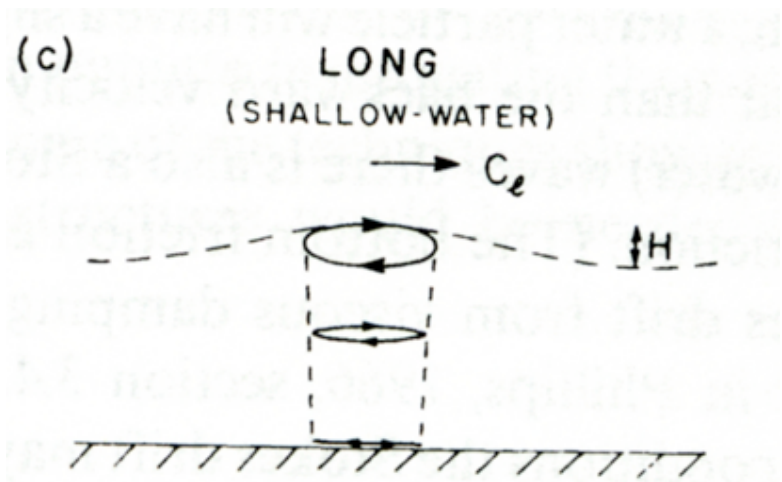
$\omega = \dfrac{2\pi}{T}$ (Angular Velocity) [1/s] (strictly [rad/s])

Celerity: c = L/T

Amplitude can be described equation: $a(x,t) = A\cos(kx - \omega t)$

Long vs. Short Waves

We maintain our small amplitude approximation and look at the limits of h/L

L << h ("short or deep wave")	L > h ("long or shallow wave")
Wave cannot feel bottom Orbits are circular Dispersive: speed dependent on wavelength  	Wave interacts with bottom (can suspend sediment) Non-dispersive Orbits are elliptical (flat) 

Dispersion, Celerity, and Energy

$\omega^2 = gk\tanh(kh)$	General dispersion relationship for surface gravity waves
$\omega^2 = gk$	Deep water wave limit (h >> L)
$\omega^2 = gk^2h$	Shallow water wave limit (h < L)

$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$	Deep water celerity: depends on properties of the wave (wavelength)!
$c = \frac{\omega}{k} = \sqrt{gh}$	Shallow water wave celerity: depends only on the medium (square root of depth)

$c_g = \frac{\partial\omega}{\partial k}$	Group Velocity, This is the speed at which Energy is carried!
$c_g = \frac{g}{2\omega} = \frac{c}{2}$	Group velocity is 1/2 the wave celerity for deep water waves
$c_g = \sqrt{gh} = c$	Group velocity and celerity are the same for shallow water waves.

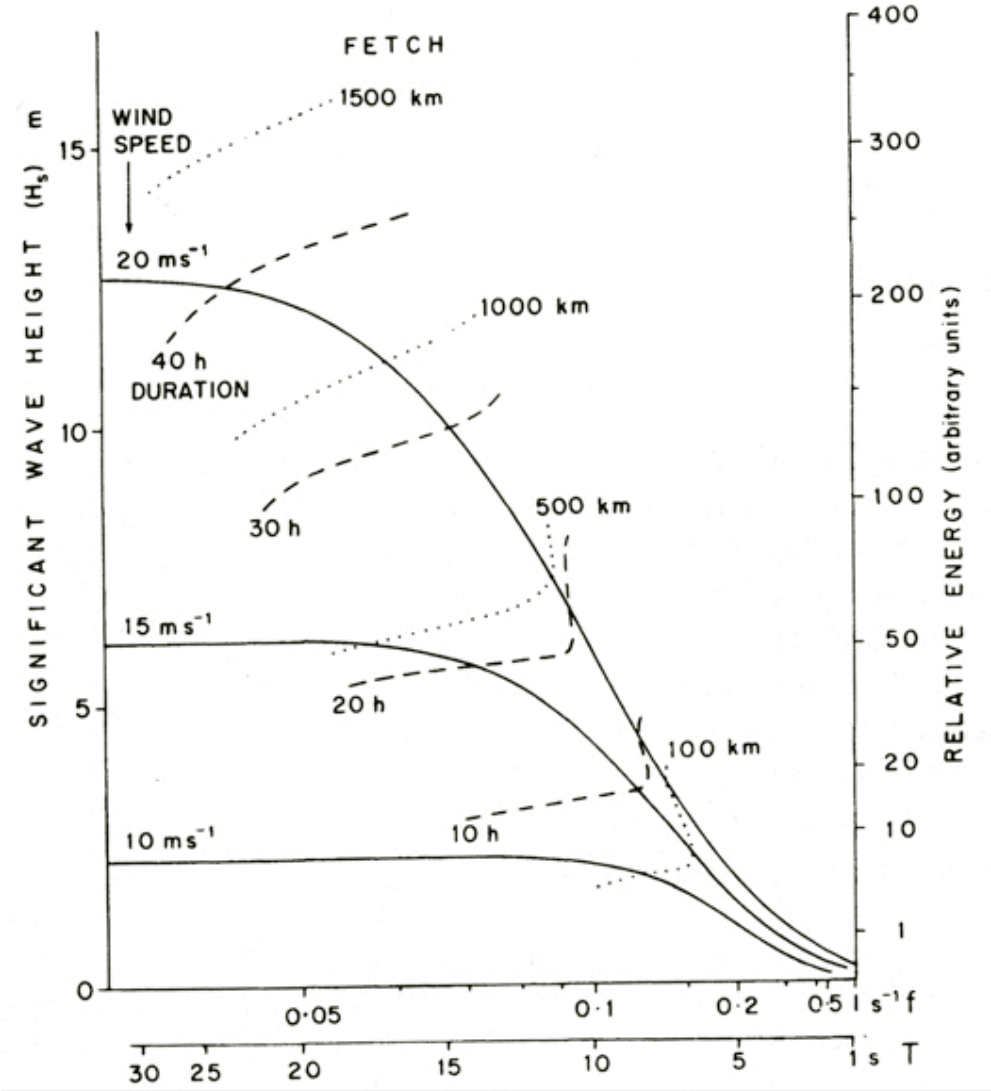
Short (Deep) Surface Gravity Waves: Wind-Driven

Important Variables:

- Wind Stress
- Time
- Fetch

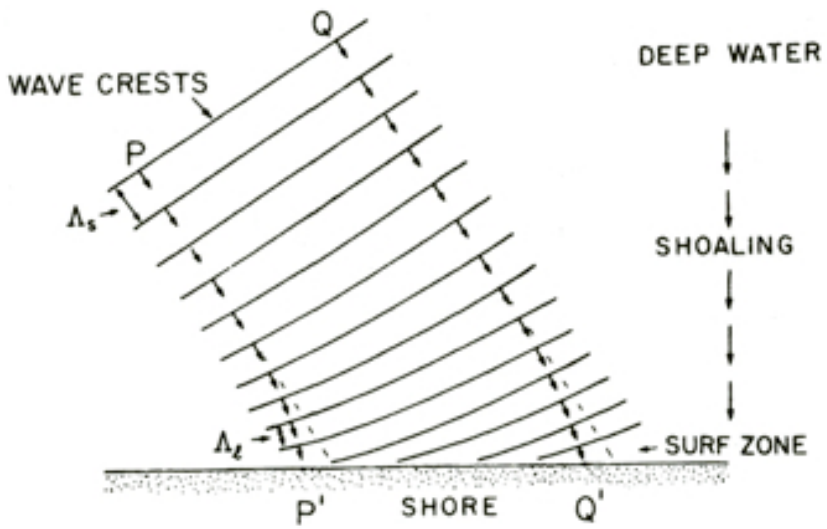
Note

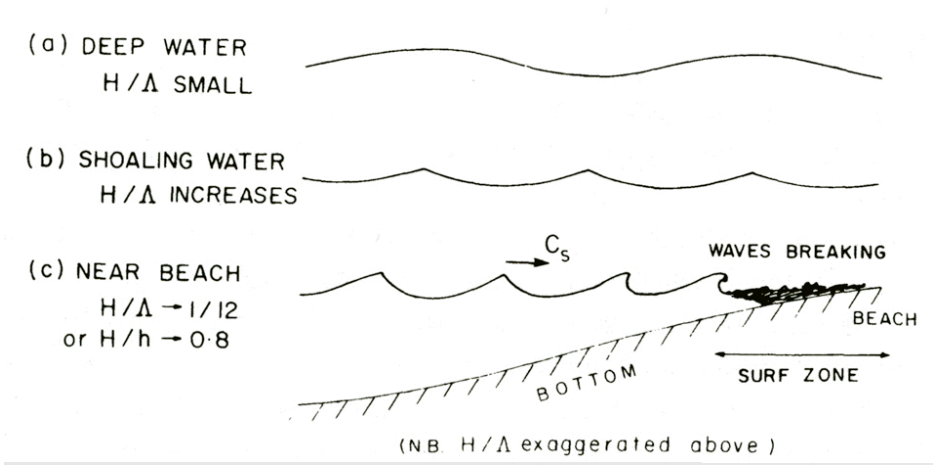
- Amplitude Asymptotes
- Lakes have short waves
- Speed/Stress relationship is very complex
- Need statistics to describe (Hsig, Tp)



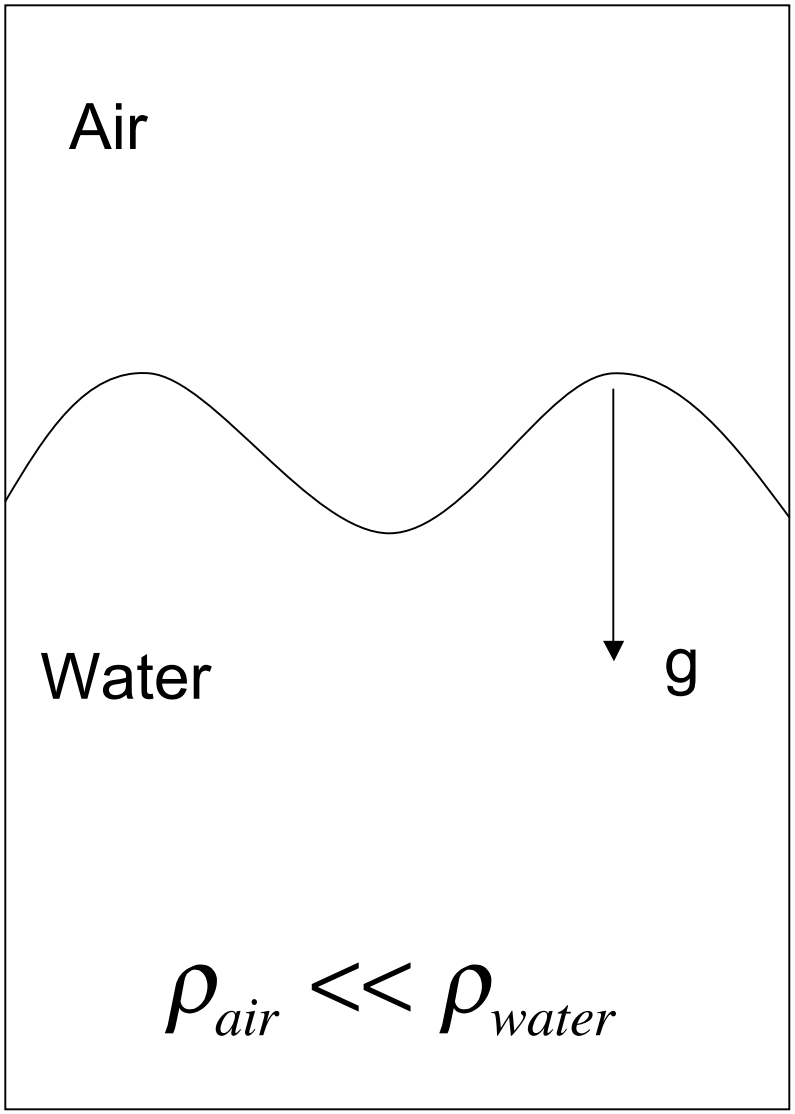
At the Coast

Refraction, Shortening, Steeping, Breaking

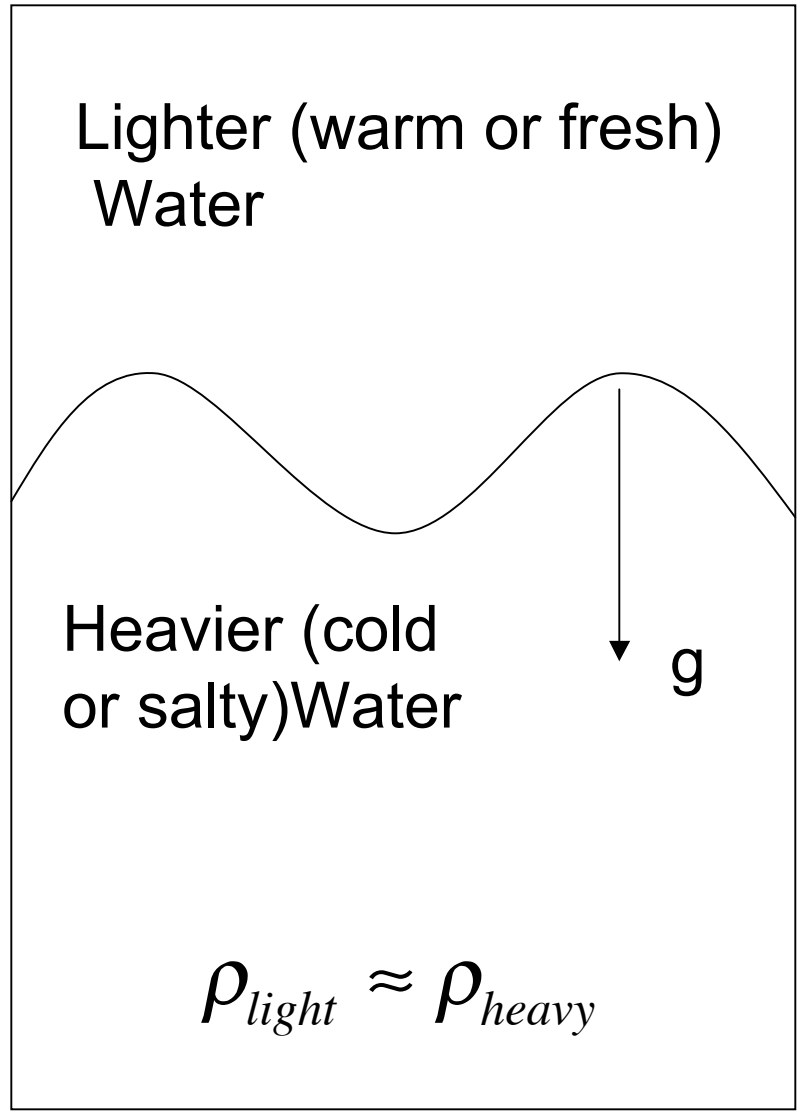




Interfacial Waves: Comparison



Surface Gravity Wave



Internal Wave

Short Internal Waves

General Dispersion Relation

$$[\omega^2 - gk\tanh k(h_1 + h_2)]\left[\omega^2 - \frac{(\rho_2 - \rho_1)gk}{\rho_2 \coth kh_2 + \rho_1 \coth kh_1}\right] = 0$$

Limiting Case 1: Short (Deep Waves)

$$\lambda_i / h_1 < 1 \qquad \lambda_i / h_2 < 1$$

$$\omega_i^{\,2} = \left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_2}\right] gk; \qquad C_i^{\,2} = \frac{g\Delta\rho}{k(\rho_1 + \rho_2)}$$

-Short Internal Waves are Dispersive

-Compared to Surface Gravity Waves they are very Slow!

$$for \quad \frac{\Delta\rho}{\rho} = 2e^{-3}: \quad \frac{C_i}{C_s} = \sqrt{\frac{\Delta\rho}{\rho_1 + \rho_2}} \cong 1/30$$

Long Waves

Waves are Not Dispersive (Analogous to Long Surf Waves)

$$\omega^2 \cong \left(\frac{gk^2\Delta\rho}{\rho_2}\right)\left(\frac{h_1 h_2}{h_1 + h_2}\right)$$

What about a thin upper layer and deep lower layer

$$C_i^{\,2} = \frac{g\Delta\rho h_1}{\rho_1}$$

Just like a surface gravity wave in depth h_1 with a speed reduction of

$$\sqrt{\frac{\Delta\rho}{\rho_1 + \rho_2}}$$

Example: S_1=0,S_2=30,T_1=T_2=10,h_1=5m: $\Rightarrow$**C_i = .8m/s**

Reduced Gravity

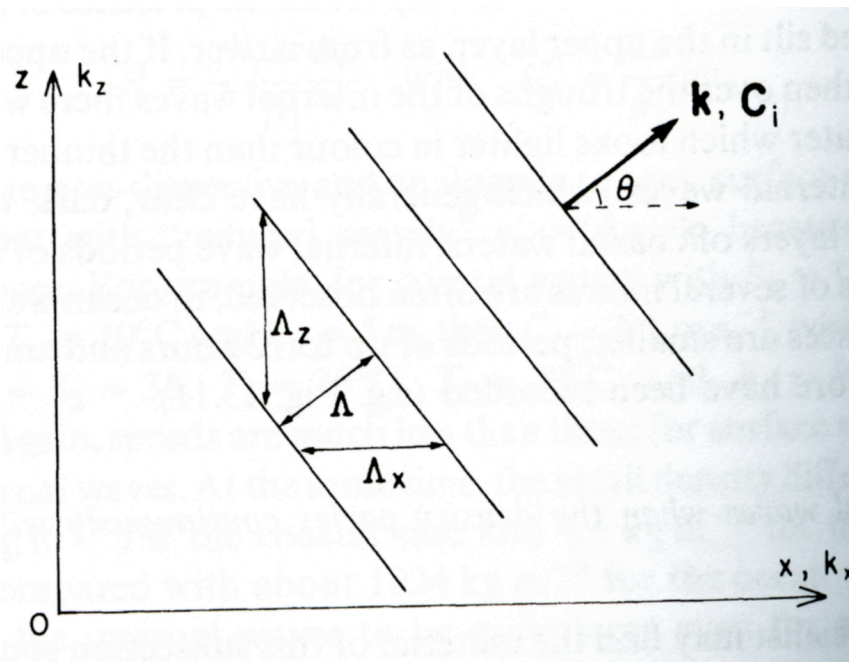
The key parameter in these relations is the reduced gravity g^*

$$g^* = g\frac{(\rho_2 - \rho_1)}{\rho_2}$$

Celerity $$\frac{C_i}{C_s} = \sqrt{\frac{\Delta\rho}{\rho_1 + \rho_2}}$$ For both Long and Short

Energy $$E = \frac{1}{2}\rho g^* A^2$$

Internal Waves in Continuously Stratified Ocean:



N is the upper limit on the wave frequency!

Brunt-Väisälä Frequency (N)

$$N^2 \cong \left(-\frac{g}{\rho}\right)\left(\frac{\partial \rho}{\partial z}\right)$$

Frequency $\omega = N \cos\Theta$

Celerity $C_i = \dfrac{N\cos\Theta}{k}$

<u>Depends on:</u>

- Properties of the medium (N)
- Wavenumber (is dispersive)
- Direction!

When is Rotation important

Acceleration:

U varies as $\sin(2\pi t / T)$

$$\frac{\partial u}{\partial t} \simeq \frac{2\pi}{T} u$$

Coriolis:

$$fu$$

When wave period approaches pendulum half-day..

$$T \simeq 2\pi / f$$

Coriolis and Acceleration are of same order!!

Kelvin Waves:

Inviscid/Linear N-S Equation + rho/h constant+ boundary

Cross-shore Momentum Equation:
Geostrophic Balance

$$g\frac{\partial \eta}{\partial y} = -fU$$

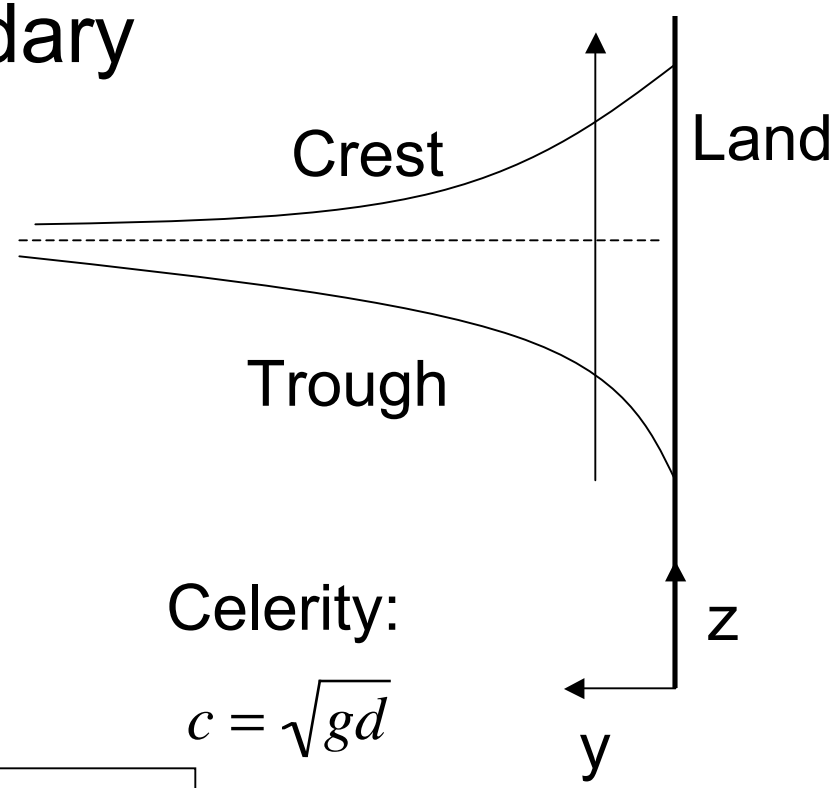
The general solution for the amplitude is:

$$\eta = \frac{H}{2} e^{-fy/C} \cos(kx - \omega t)$$

Velocity:

$$U = \frac{H}{2}\frac{C}{d} e^{-yf/C} \cos(kx - \omega t); V = 0$$

Decay Scale $C/|f|$

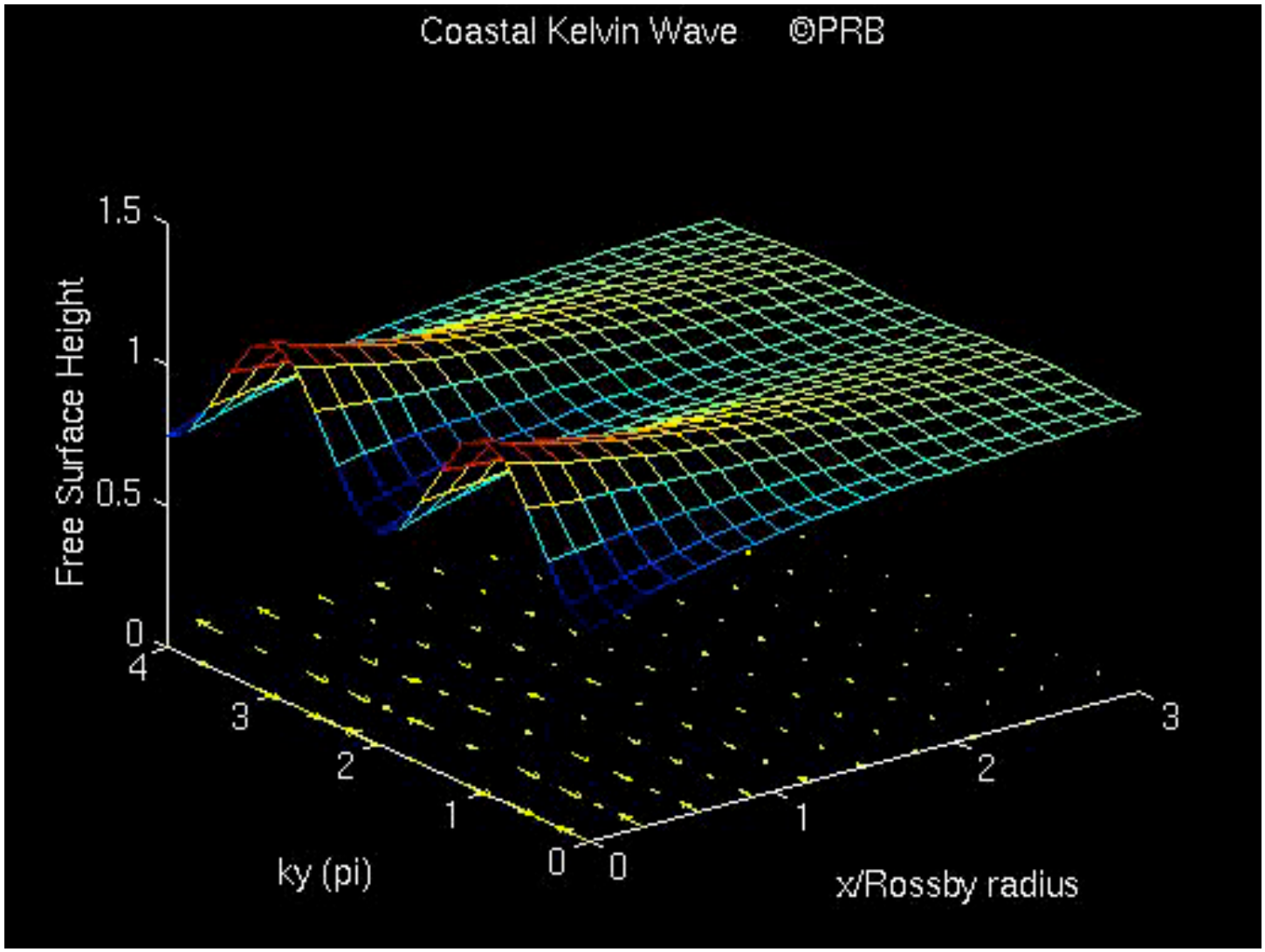


Celerity:

$$c = \sqrt{gd}$$

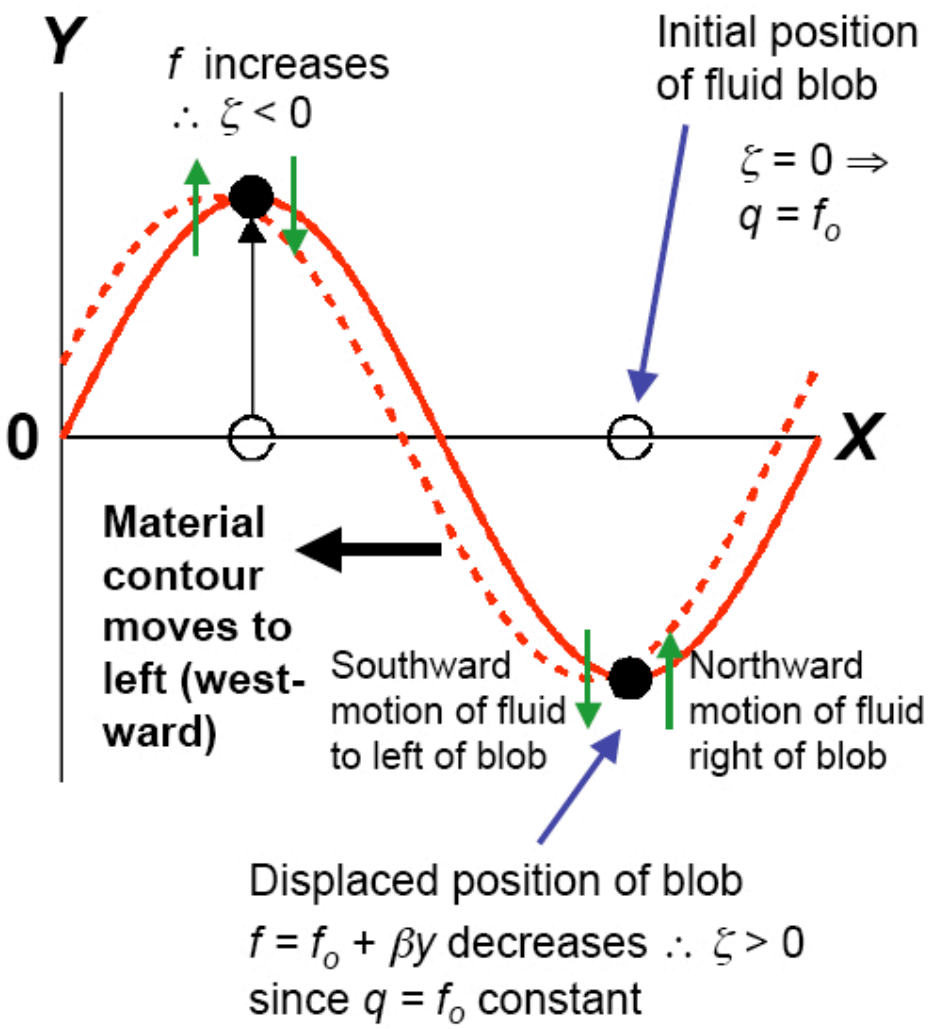
Solution first derived by Lord Kelvin: Is is a Kelvin Wave!

Structure



Credit?

Rossby (Planetary) Waves



McLandress

Key is conservation of potential vorticity $PV = \frac{\xi + f}{D}$

Consider some initial perturbation around a line of constant latitude.

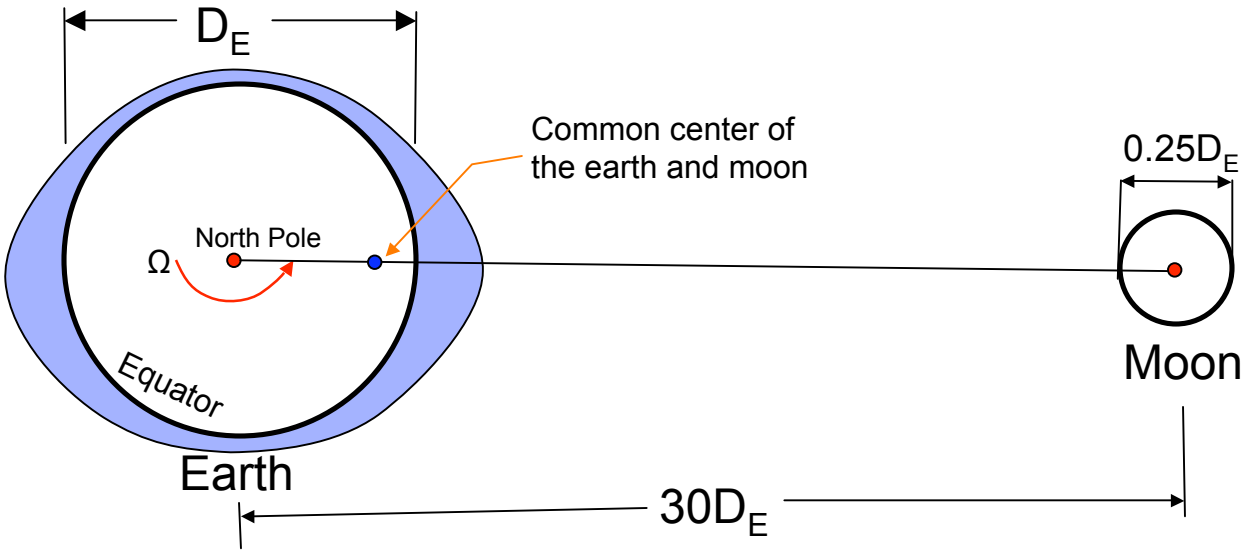
Restoring force is generated supported westward wave propagation.

Note that changes in D can also support waves (Topographic Planetary Waves)

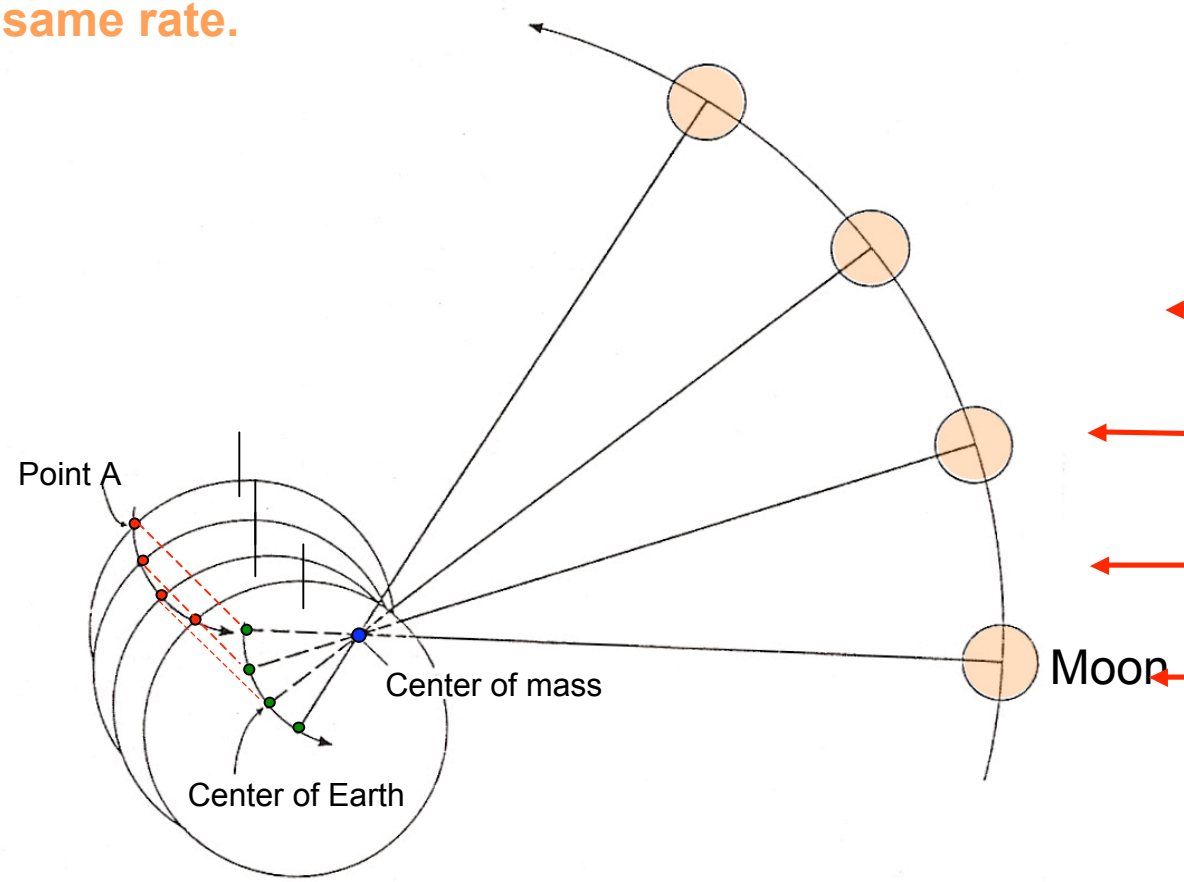
Astronomically Forced Long Surface Gravity Waves: Tides

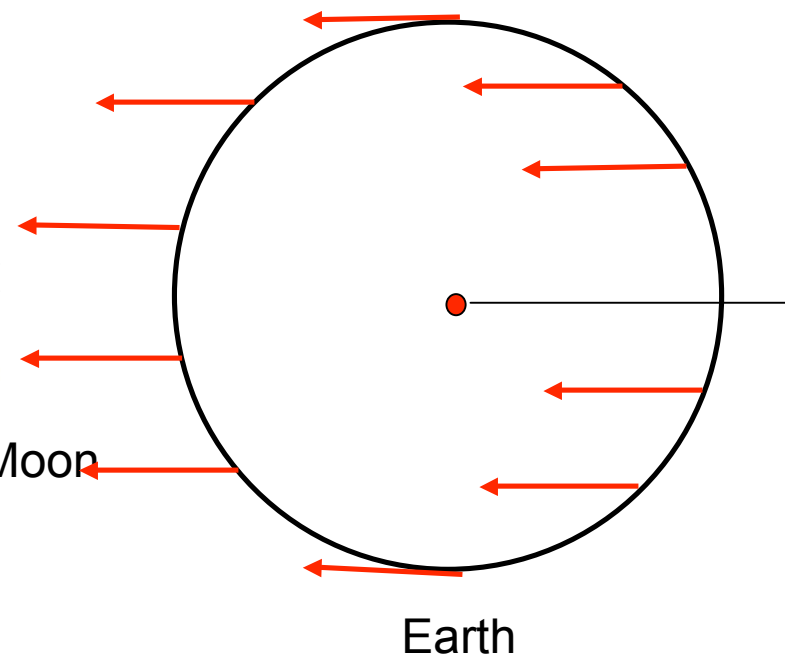
The tide is the name given to the periodic rise and fall of sea level. This is one of the earliest scientific venture of the oceanic exploration.

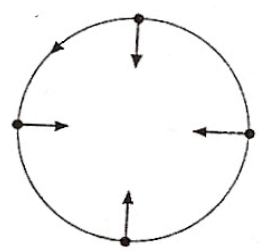
Causes of Tides (for the global scale): Gravitational pull of the moon and sun



If we stop the Earth from spinning on its axis relative to an inertial reference frame and let the moon continue in its normal orbit, we find that every point on the Earth rotates around the center of mass point at the same rate.



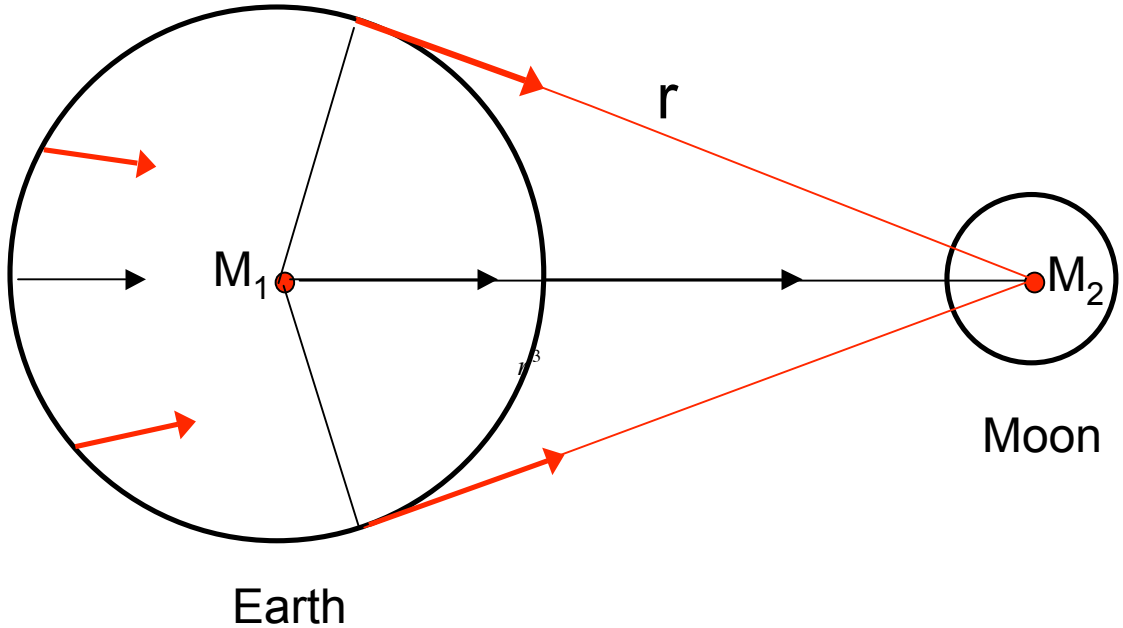




Centrifugal
Force is Constant

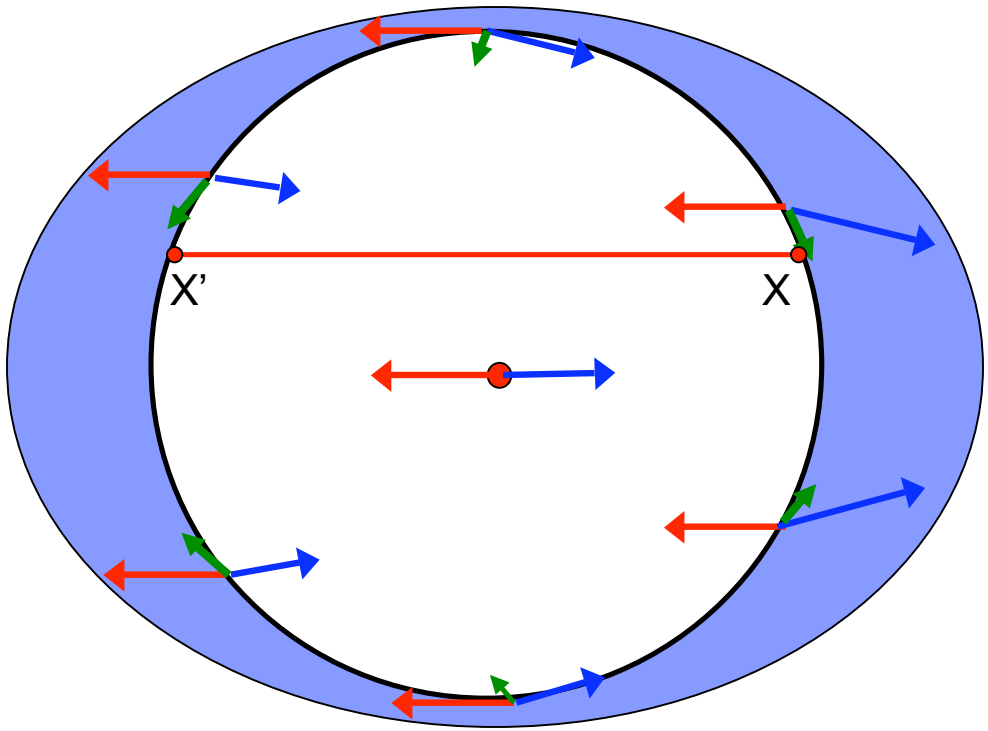
Gravitational Force ($\vec{F}_g$):

$$\vec{F}_g = \mu \frac{M_1 M_2}{r^2} \left(\frac{\vec{r}}{r}\right)$$



The gravitational force is larger on the moon side than the opposite side of the moon due to the differences in distance.

Sum of gravitational and centrifugal forces



x and **x'** indicate the change in position of a point on Earth's surface after half a lunar day (12 hours 25 minutes), which have two equal tides per 24 hours and 50 minutes at these two positions.

Semidiurnal tide: period-12.42 hours

At the center of the earth:

$$\left|\vec{F}_c\right| = \left|\vec{F}_g\right|$$

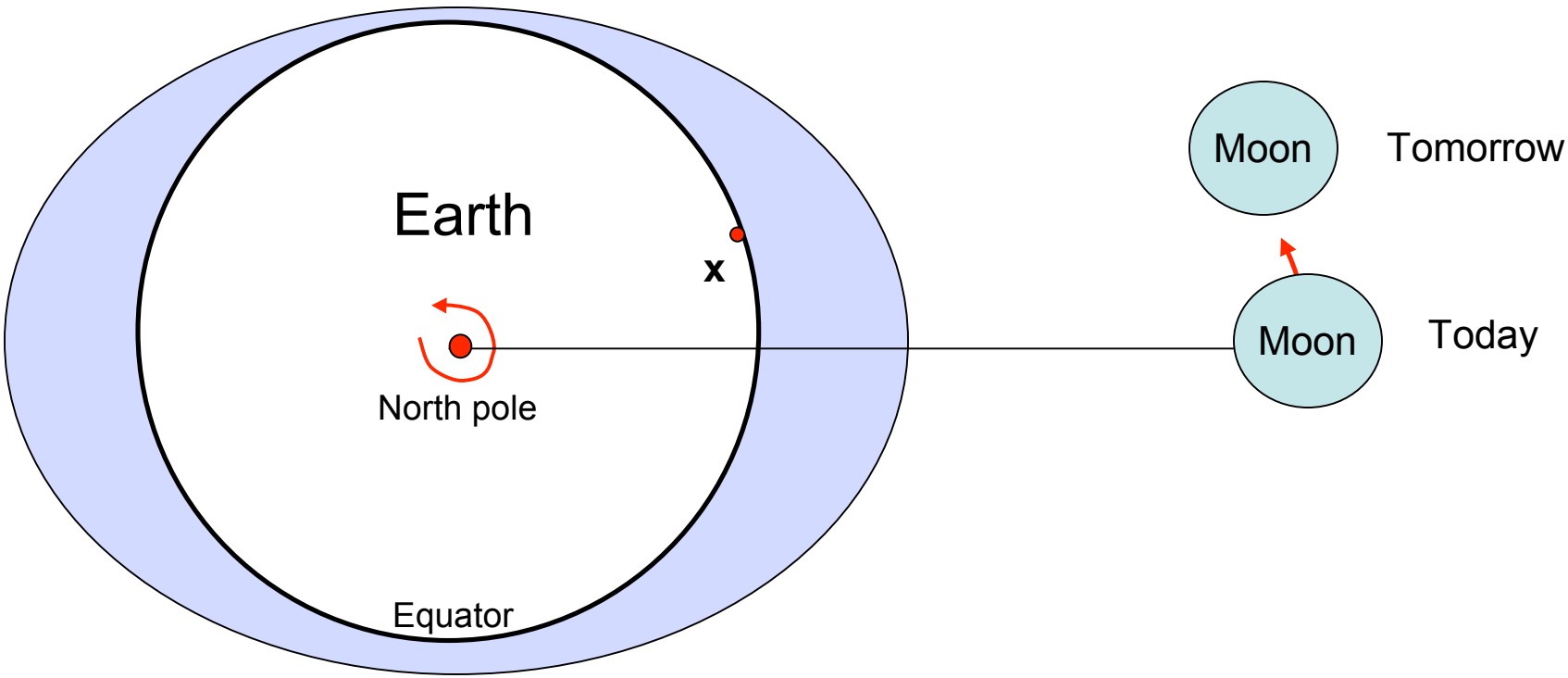
On the side facing to the moon:

$$\left|\vec{F}_c\right| < \left|\vec{F}_g\right|$$

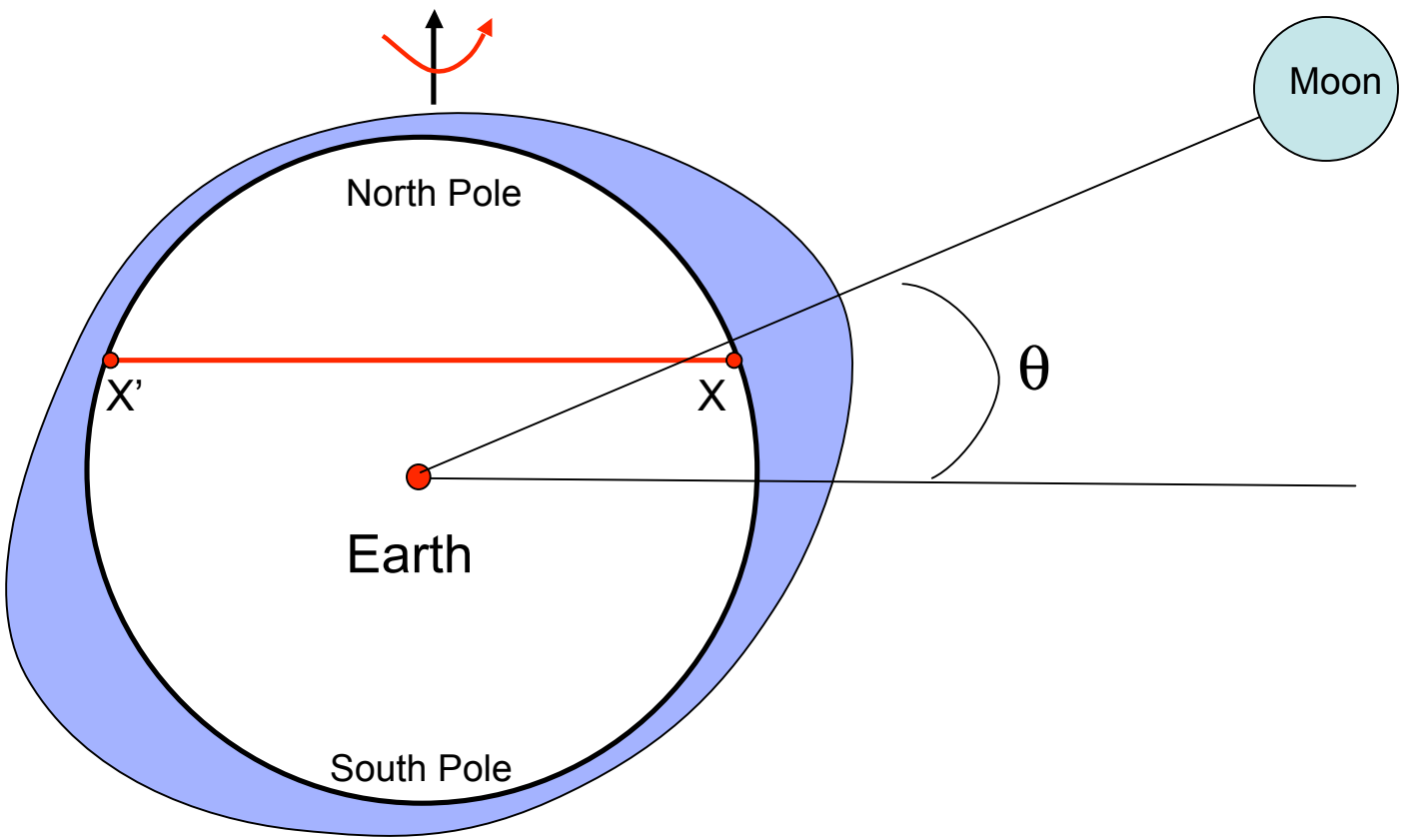
On the back side to the moon:

$$\left|\vec{F}_c\right| > \left|\vec{F}_g\right|$$

Top view from the north pole

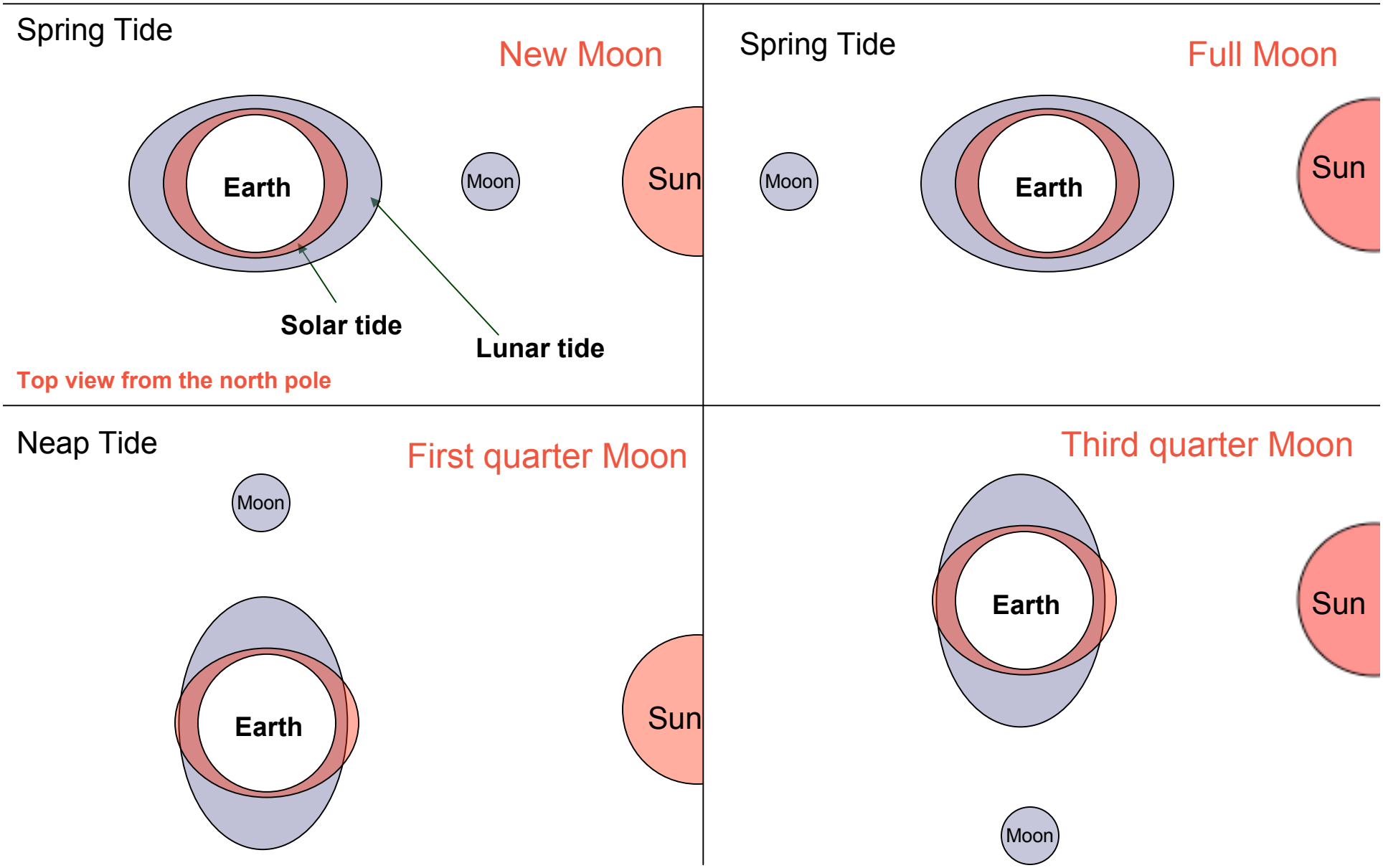


The moon's location relative to the earth varies with time. The one-day movement of the moon cause the delay of high water or low water timing by approximately 50 minutes.

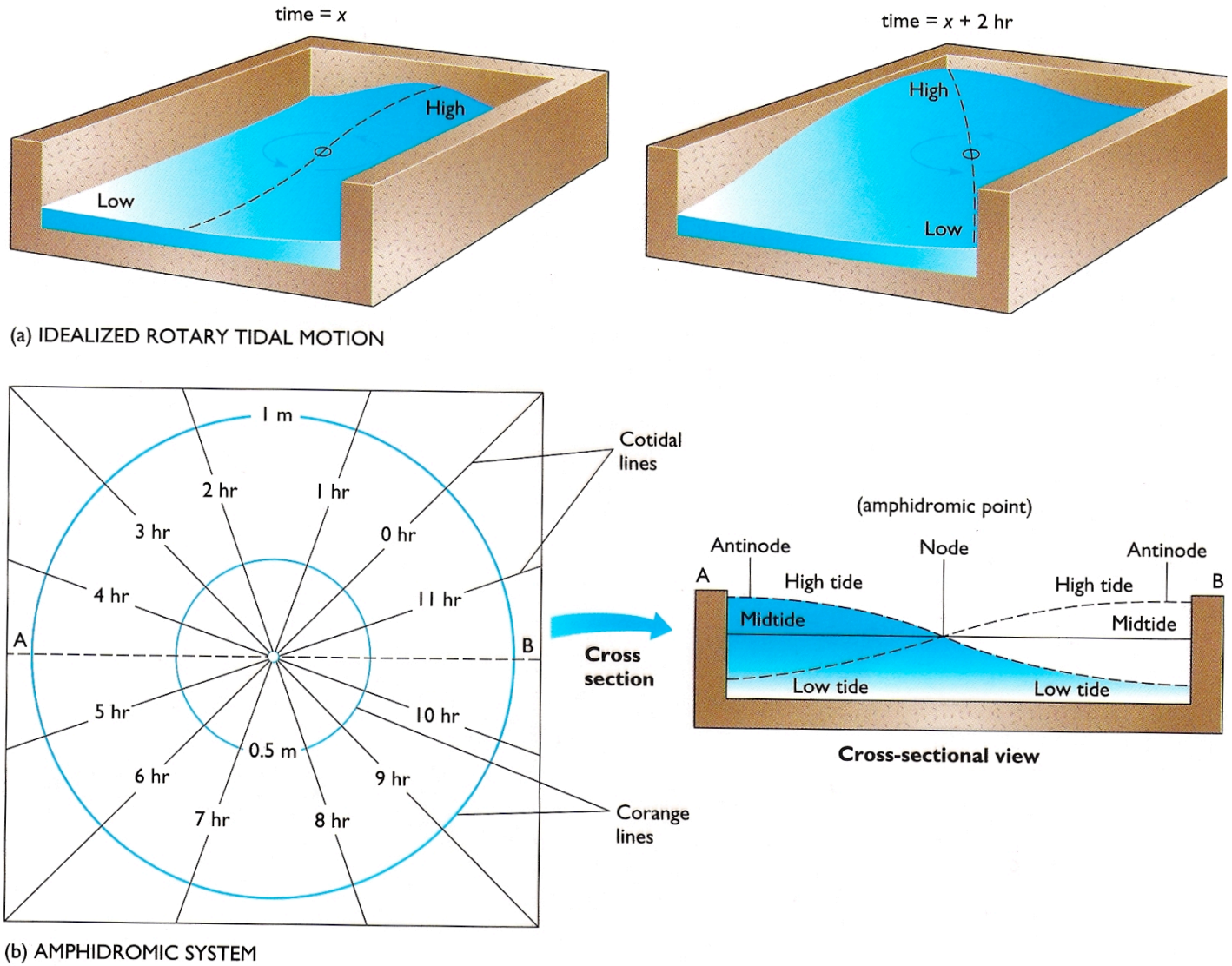


The moon moves at various angles to the north and south of the equator up to a maximum angle of 35° (depending on the season and also the time of the lunar month. Then observer at the location of "x" will notice two high tides with unequal height per lunar day. It is called "**diurnal inequality**".

Spring and Neap tides [fortnightly (14-days) variation]

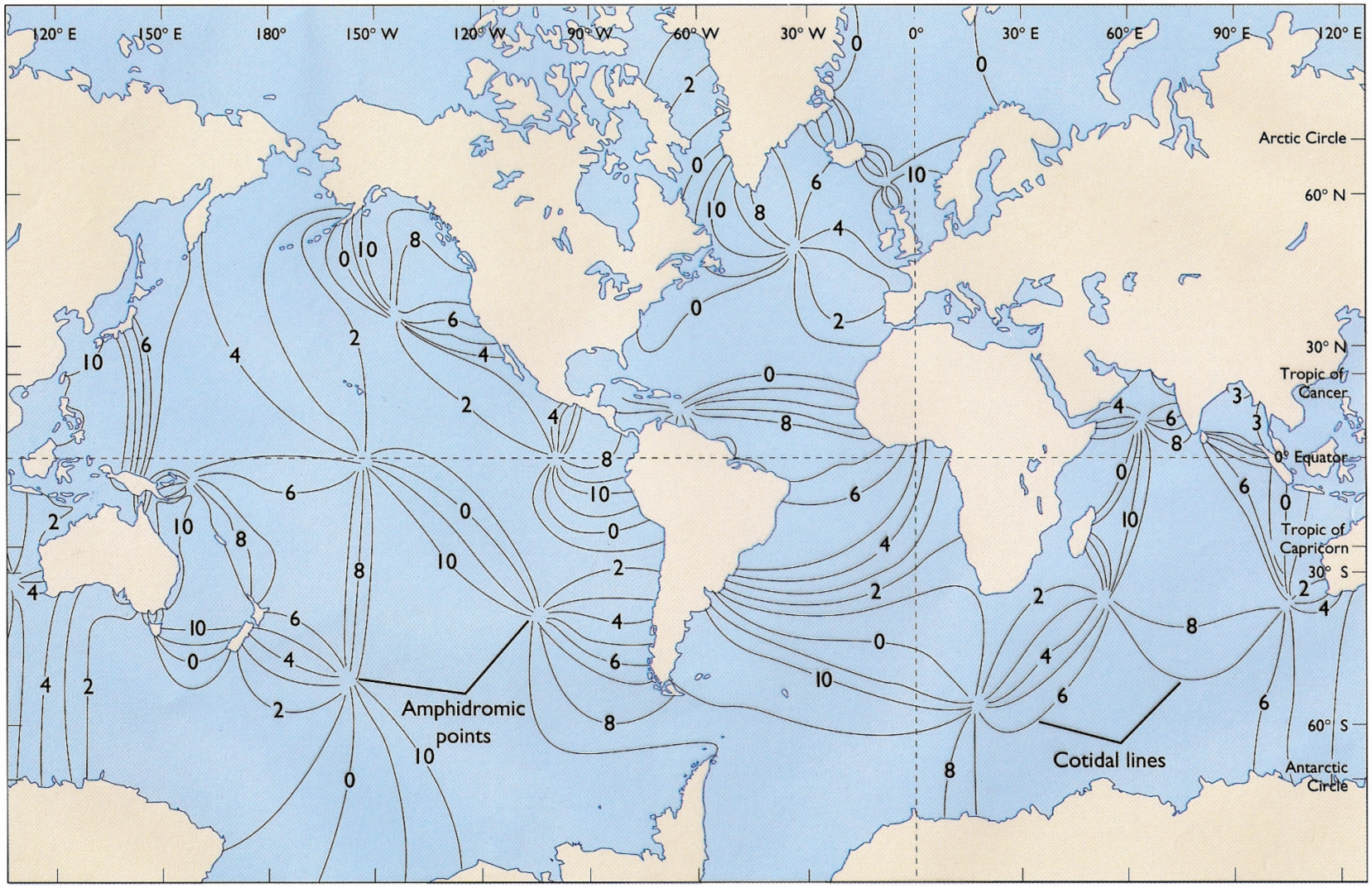


Amphidromic System



Acetate 24 (Figure 7-7)

Global amphidromic systems



Acetate 25 (Figure 7-9)

Tidal Terminology

High water (or high tide):	the maximum water level during a tidal period
Low water (or low tide):	the minimum water level during a tidal period
Mean tidal level:	the mean water level relative to a reference point over a long-term average
Tidal range:	the difference between high and low waters
Daily inequality:	the difference between two successive low or high tides
Spring tide:	the tides at full and new moons
Neap tide:	The tide at first and third quarter moons

Major Tidal Constituents

The tides observed in the ocean is the sum of more than hundred harmonic periodic oscillations. In most situations, the tide is dominated by five major tidal constituents given below.

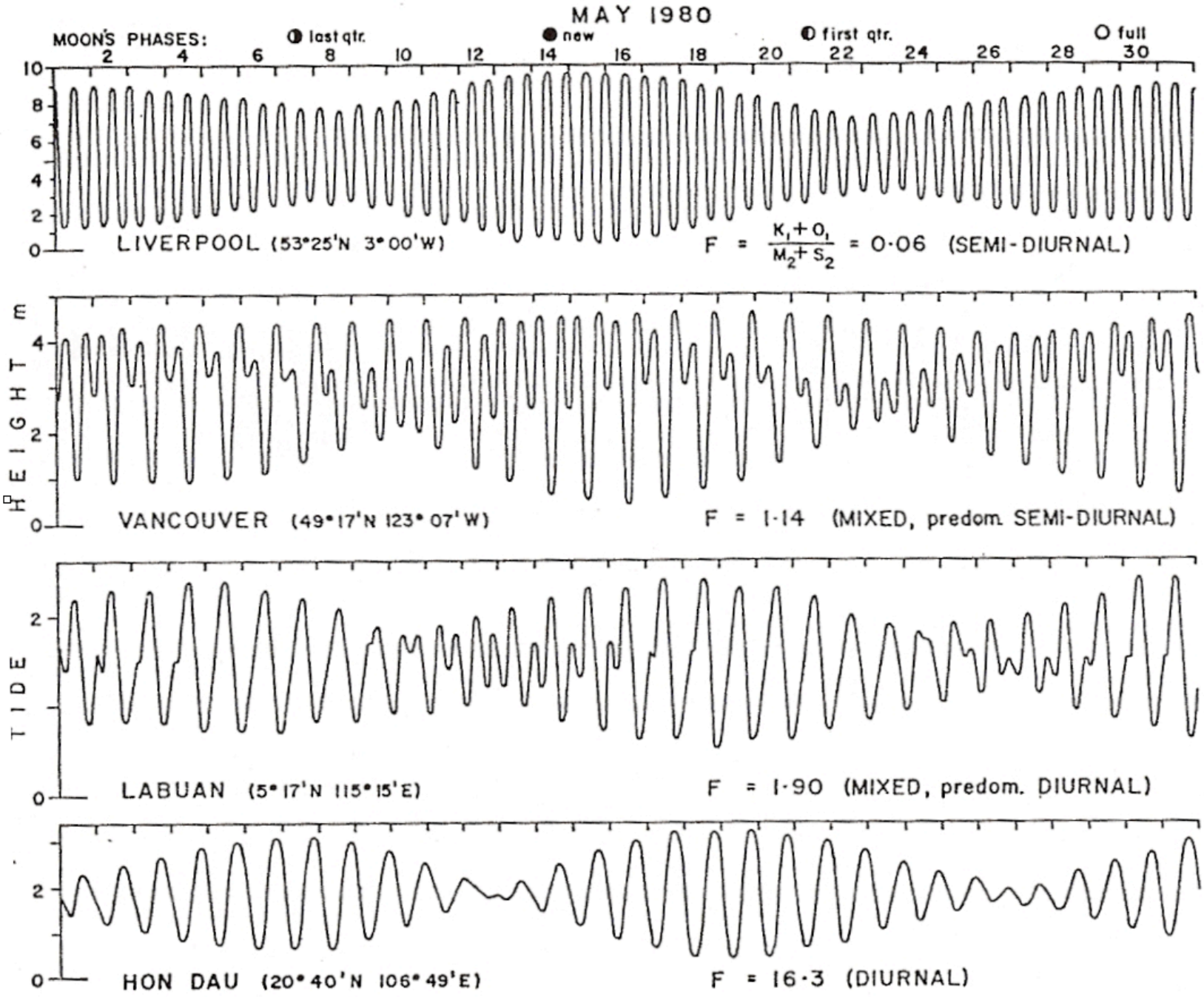
M_2 tide	12 h 25 min (1/2 lunar day)-semidiurnal lunar tide produced by the moon
S_2 tide	12 hours (1/2 solar day)-semidiurnal solar tide produced by the sun
N_2 tide	12.66 hours-larger lunar elliptic induced tide.
O_1 tide	24 h 50 min-diurnal lunar tide produced by the moon
K_1 tide	24 hours-diurnal solar tide produced by the sun

Tidal Classification

In most regions, the change of the tidal elevation and currents are dominated by either semidiurnal tides or diurnal tides. A more systematic classification of tidal types is defined by the so-called "Form Ratio" given as

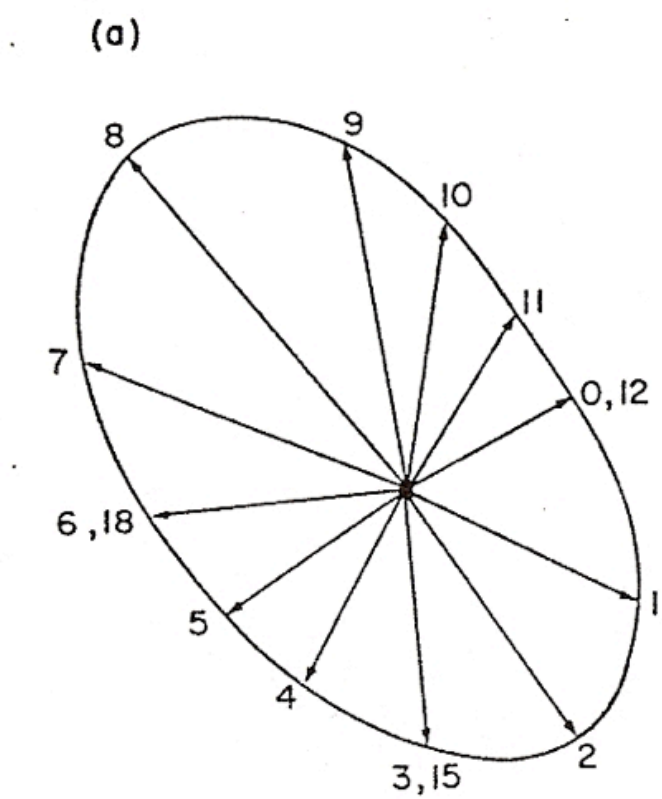
$$F = \frac{K_1 + O_1}{M_2 + S_2}$$

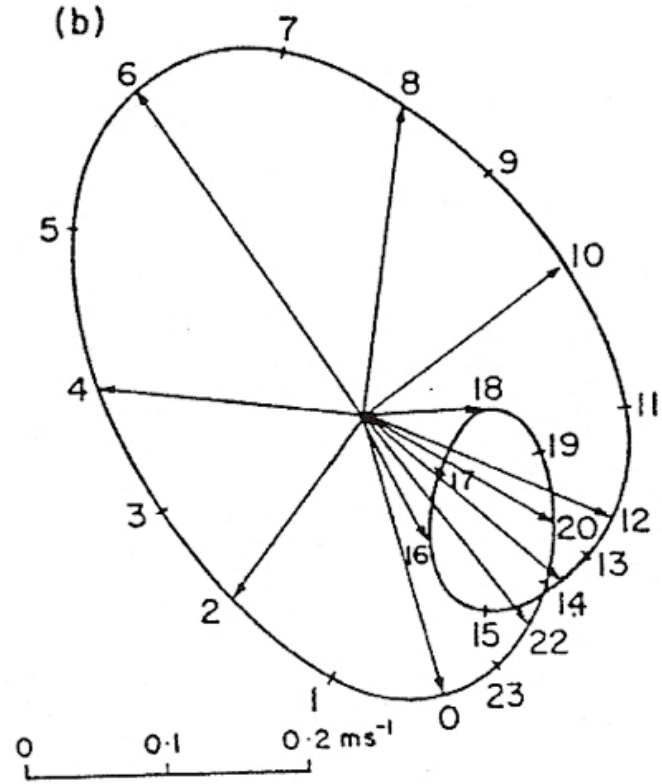
0-0.25	Semidiurnal tides: two high waters and low waters with about the same height each day. The mean spring tide range = 2 (M_2 +S_2)
>0.25 and <1.5	Mixed tides with mainly semidiurnal tides: large inequalities in tidal range and time between highs and lows each day. The mean spring tide range = 2 (M_2 +S_2)
1.5-3.0	Mixed tides with mainly diurnal tides: frequently only one high water per day. he mean spring tide range = 2 (K_1 +O_1)
> 3.0	Diurnal tides: generally only one high water per day. The mean spring tide range = 2 (K_1 +O_1)


MAY 1980
MOON'S PHASES:
last qtr.
new
first qtr.
full
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
HEIGHT m
LIVERPOOL (53°25'N 3°00'W)
F = (K1+O1)/(M2+S2) = 0·06 (SEMI-DIURNAL)
VANCOUVER (49°17'N 123°07'W)
F = 1·14 (MIXED, predom. SEMI-DIURNAL)
TIDE
LABUAN (5°17'N 115°15'E)
F = 1·90 (MIXED, predom. DIURNAL)
HON DAU (20°40'N 106°49'E)
F = 16·3 (DIURNAL)

Tidal Currents

In the global and regional oceans, the tidal currents rotate with time over a tidal cycle.

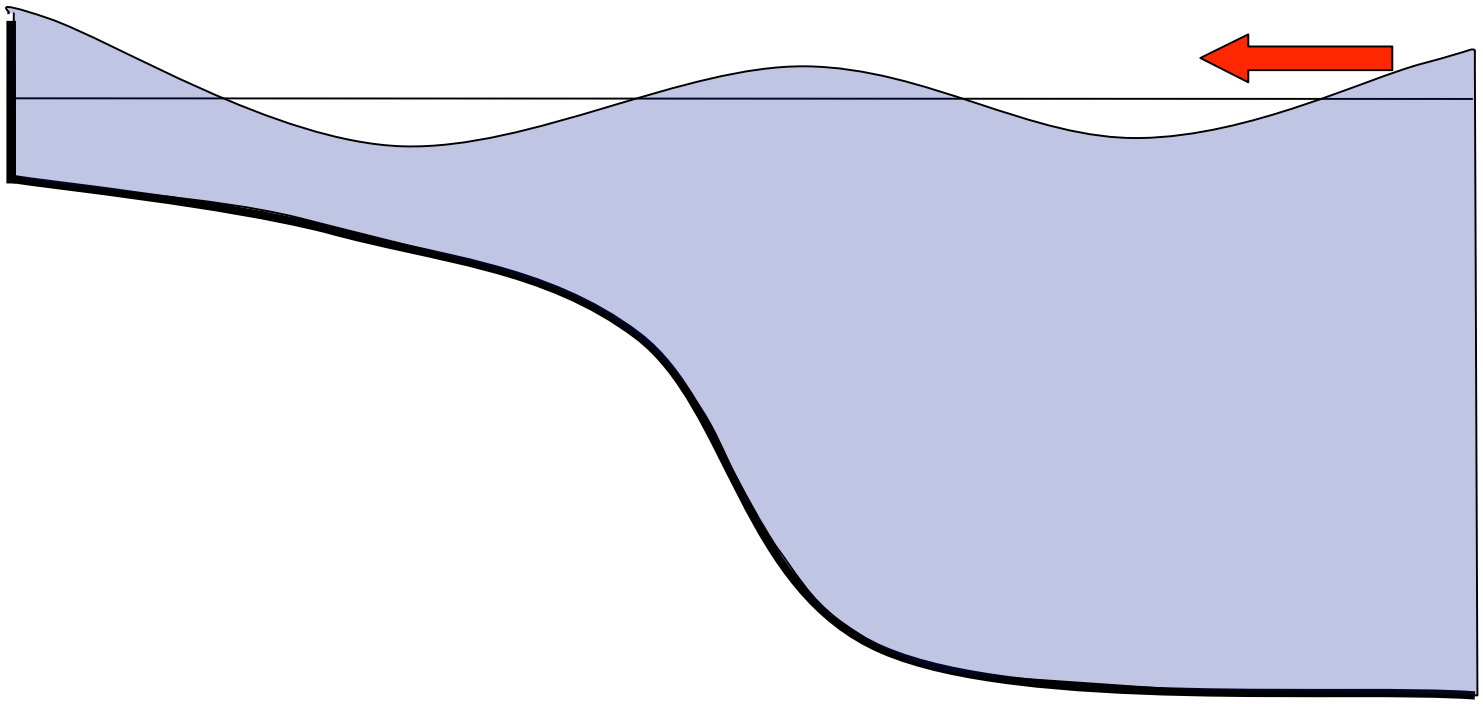




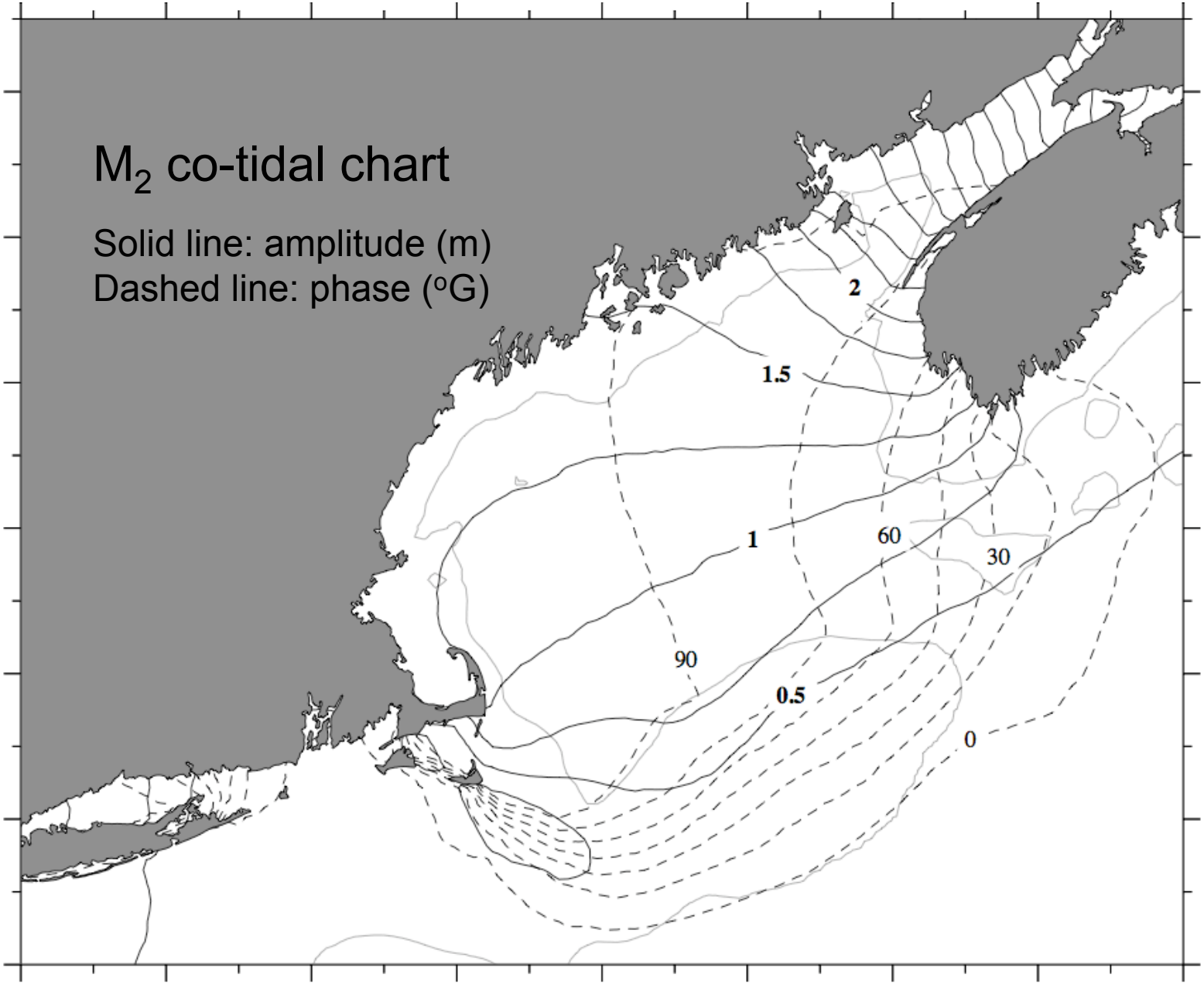
Semi-diurnal equal tide

Semi-diurnal unequal tide

The tides observed in the coastal region consists of two parts: equilibrium tides and tidal waves propagating from the open ocean.



- Strong near the coast and weak in the open ocean;
- Vary over topography in the shallow water and estuaries
- Resonance when the tidal frequency match the local geometric frequency



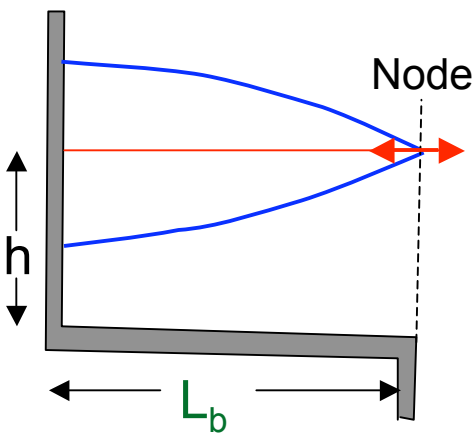
In the Gulf of Maine, the semi-diurnal equilibrium tidal amplitude ~0.38 m

Resonance solution: Open Domain

For the first case, $L=2\lambda$ and $n = 1$

$$T = \frac{2L}{\sqrt{gh}}$$

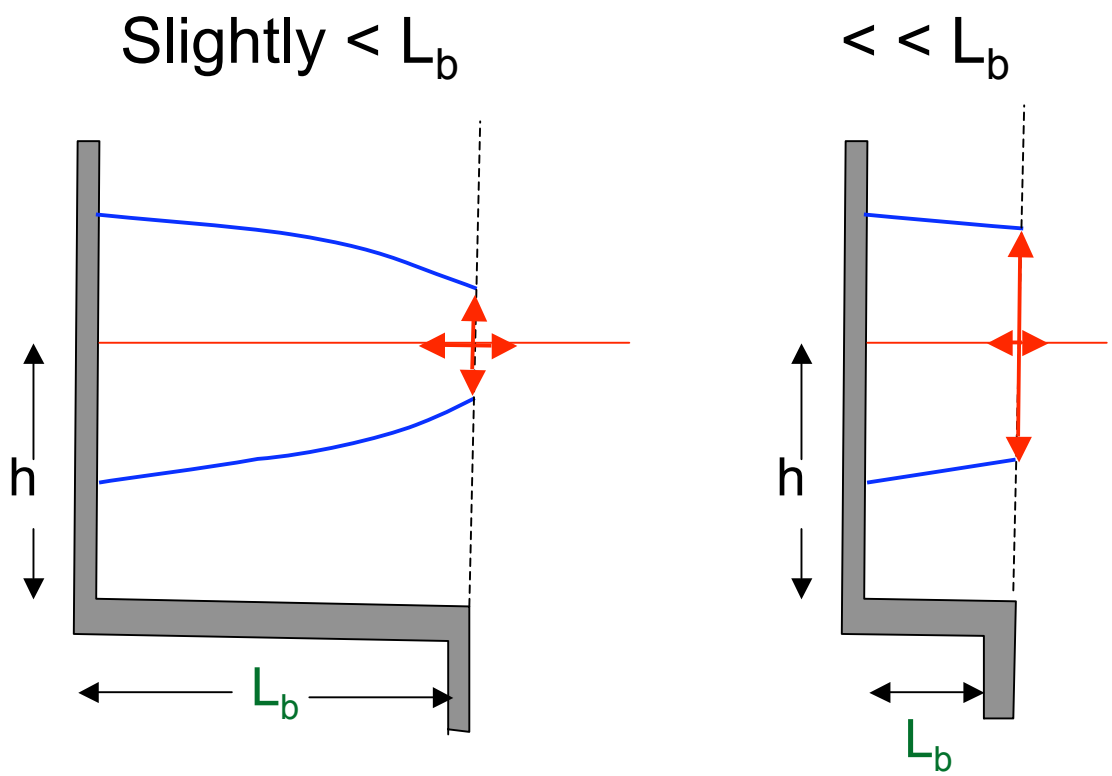
Let us consider the semi-enclosed bay in which the water flow in during the flood tide and flow out during the ebb tide. The length of the bay $L_b = L/2$



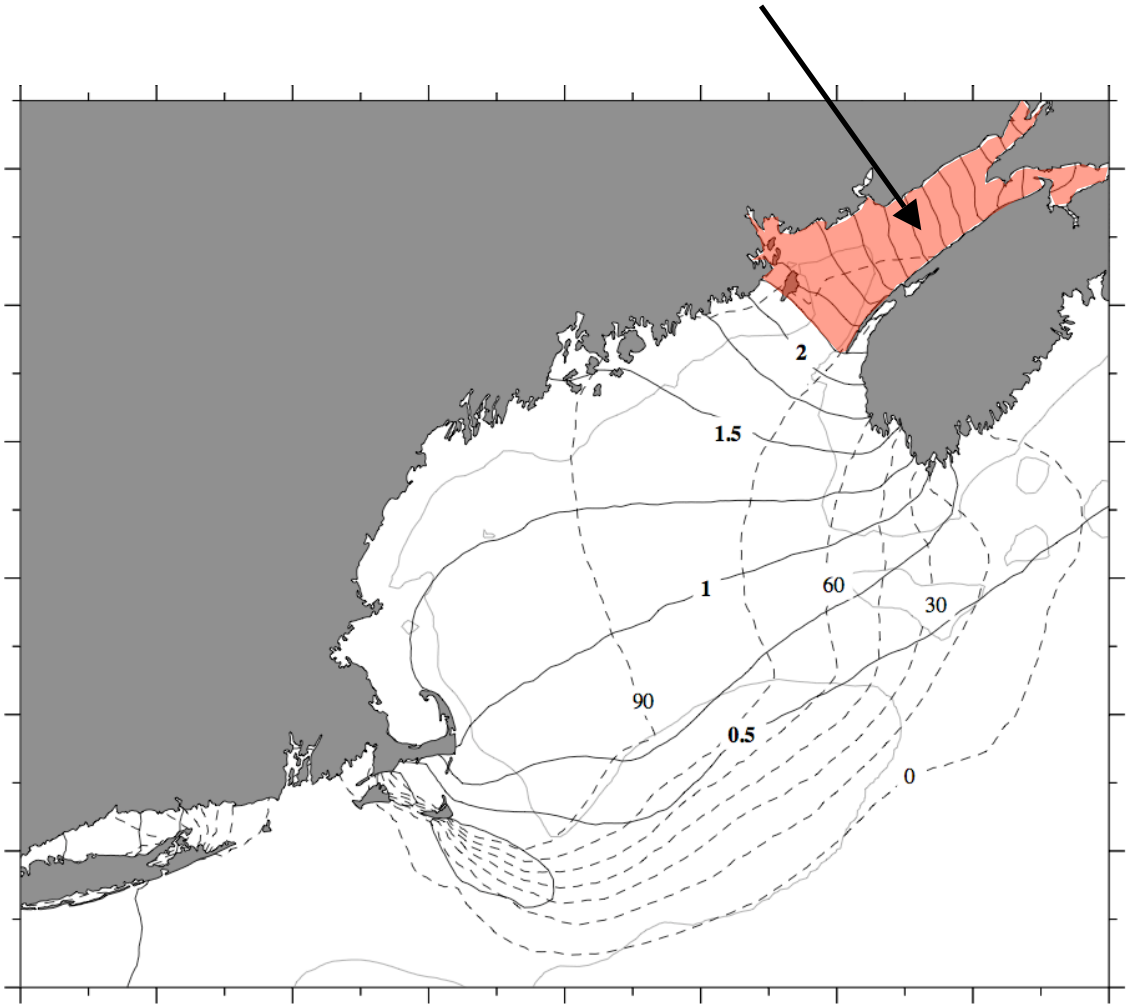
$$T = \frac{4L_b}{\sqrt{gh}}$$

: the natural period of the bay

If the period of the progressive wave entering this bay is equal to this period, the oscillation will become “resonant”


Slightly < L_b
< < L_b
h
L_b
h
L_b

> 2 m with a maximum tidal range of ~ 8 m



The lowest natural surface gravity wave mode in the Gulf of Maine (GOM) and Bay of Fundy (BF) region is

12.8 hours

The M_2 tidal wave (12.42 hours) enters the GOM/BF is near the resonance period:

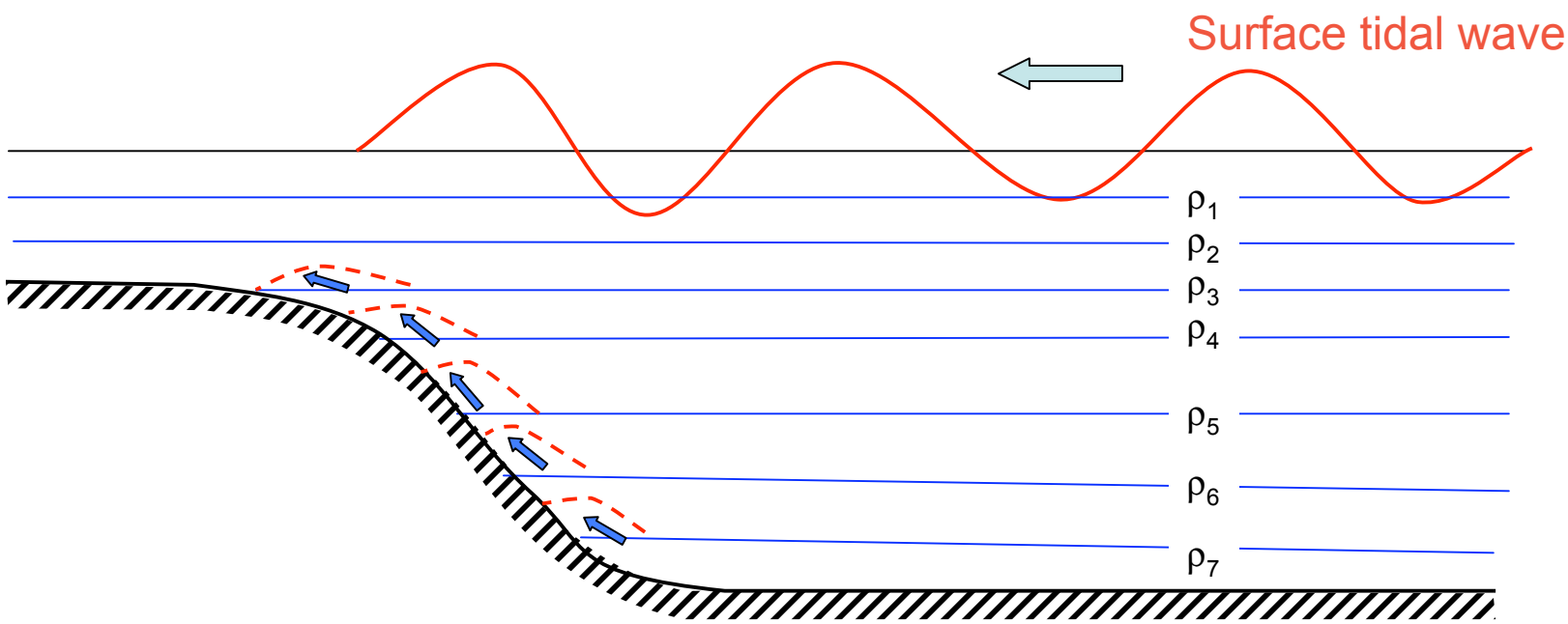
High tidal elevation !

Garrett (1972)

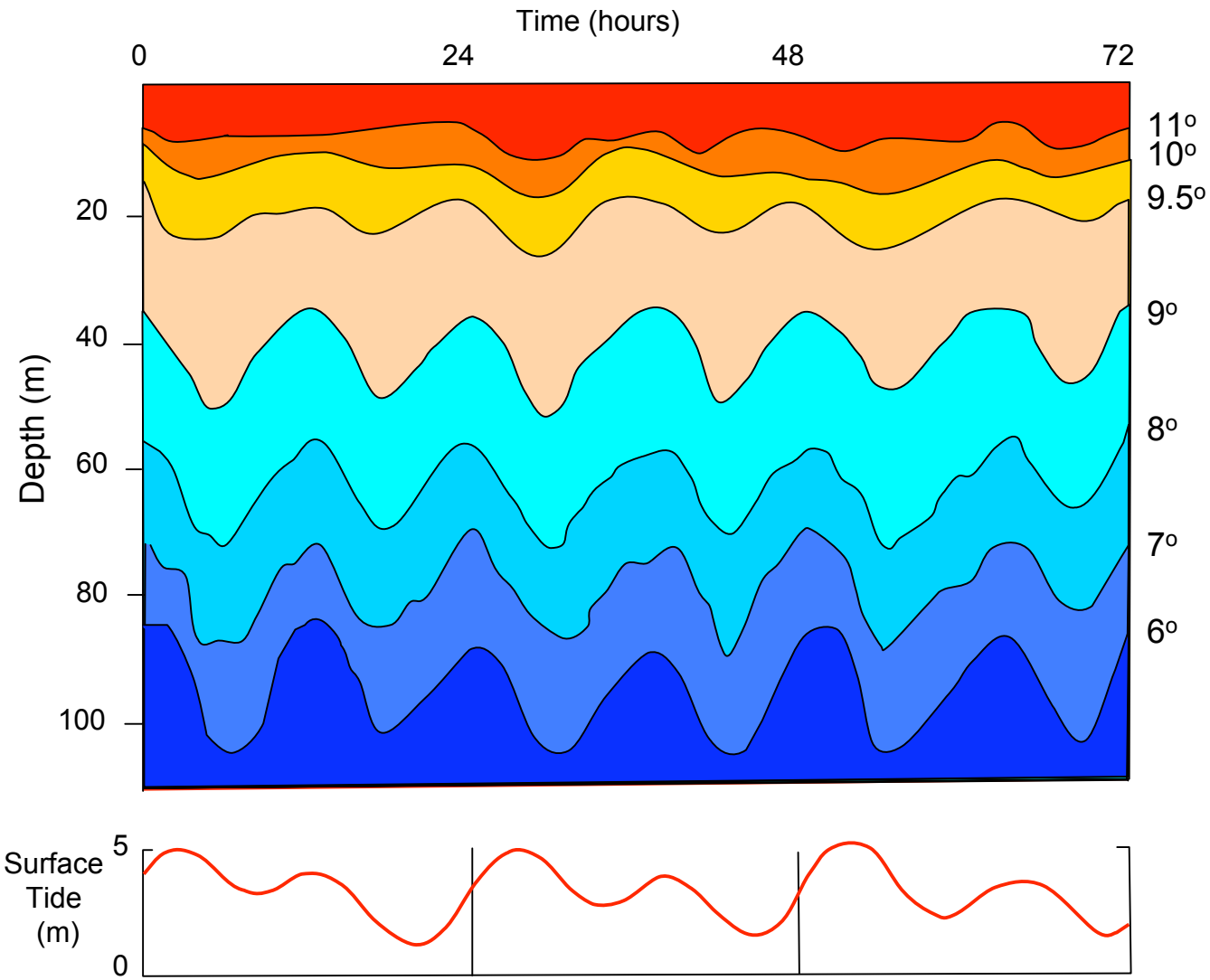
Internal Tides

Causes:

When a surface tidal wave propagates onto the slope, the interaction between the tidal currents and bottom topography can lead to the vertical oscillation of pycnoclines and hence produce internal waves with the tidal period. Such a tidal-induced internal wave is called the “internal tide”.



- Large amplitudes but slow phase speed
- Intensifies near the bottom and decreases upward



Internal tides in Bute Inlet, British Columbia, Canada in July 1953. Note that the surface tide is magnified by a factor of 4.