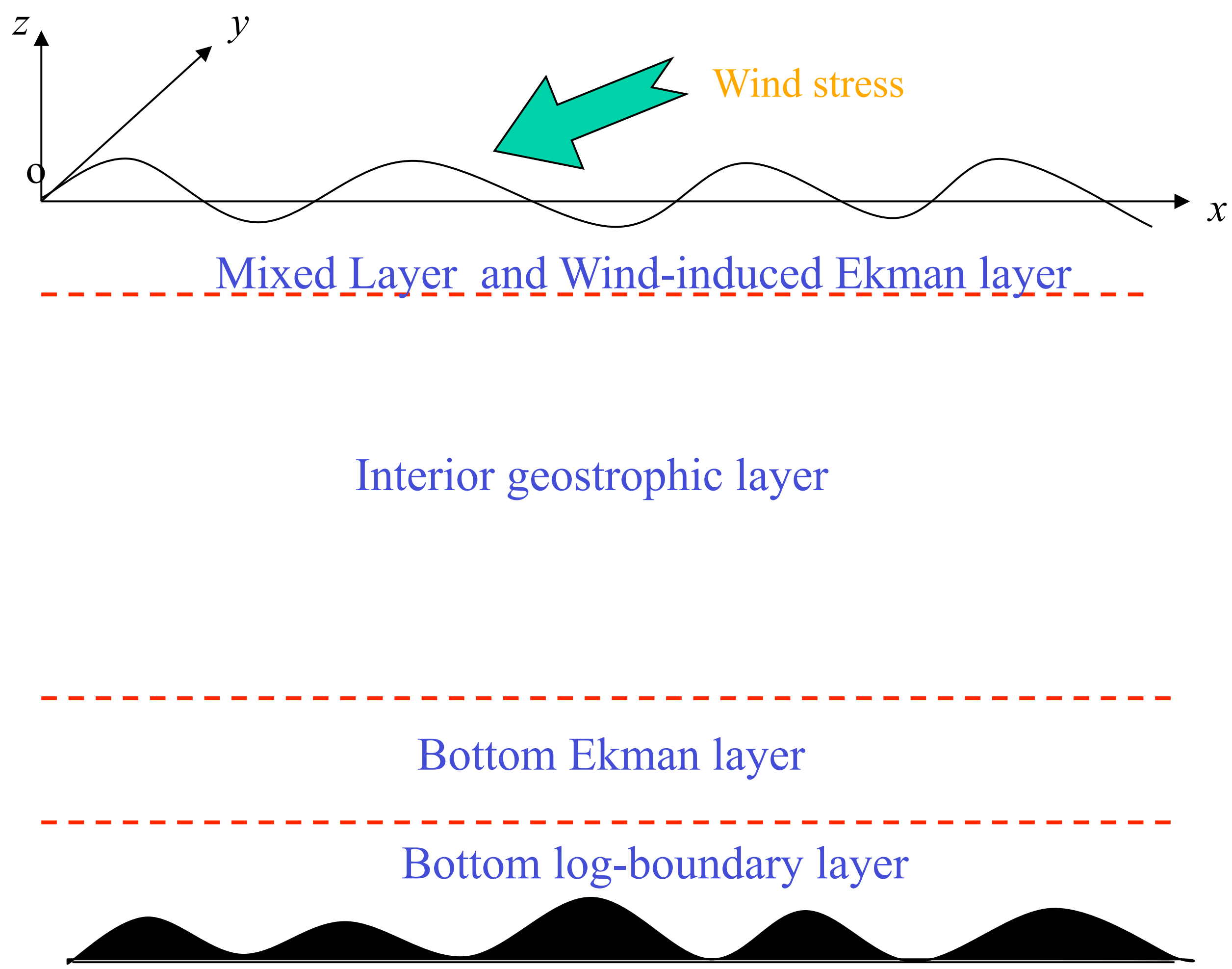
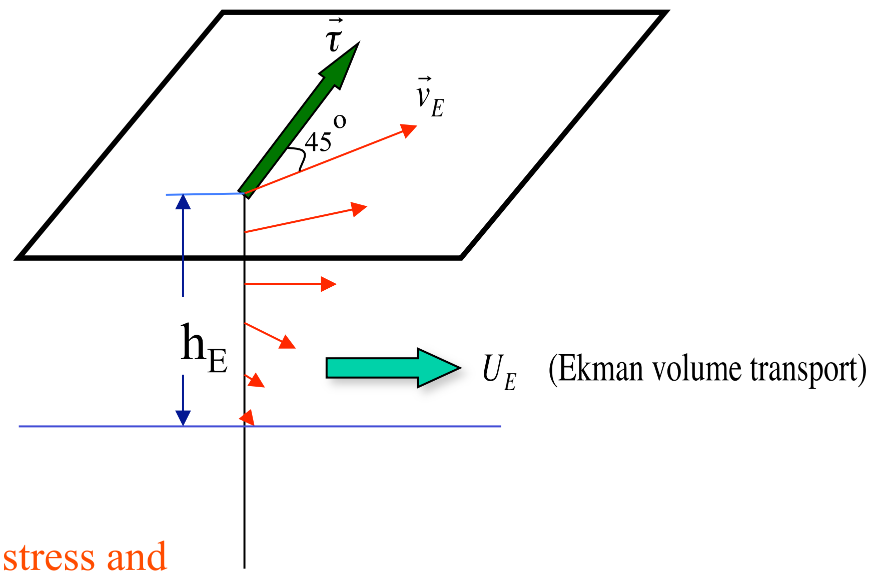
MAR650: Lecture 2: Layer Oceans and Mixed Layer Dynamics



1) Wind-induced surface Ekman layer

$$\begin{cases} -fv_E = K_m \dfrac{\partial^2 u}{\partial z^2} \\ fu_E = K_m \dfrac{\partial^2 v}{\partial z^2} \end{cases} \longrightarrow \boxed{\text{Coriolis force = Vertical diffusion}}$$



a) Current profile:

The Ekman velocity decreases and rotates clockwise with depth Ekman spiral.

b) The Ekman layer thickness (depth):

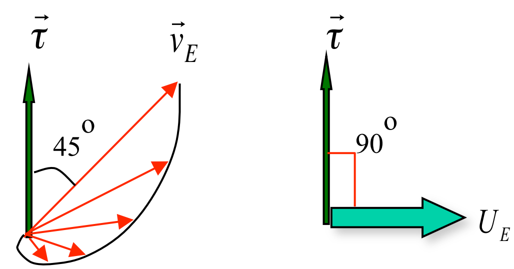
$$h_E = \sqrt{\frac{2K_m}{f}}$$

Directly proportional to turbulent viscosity coefficient and inversely proportional to the Coriolis parameter.

c) The direction of the surface Ekman current:

$$\tan\alpha = \frac{v_E}{u_E} = 1, \quad \alpha = 45^o$$

The angle between the wind stress and surface Ekman current is 45º. On the northern hemisphere, the surface Ekman current is 45º on the right of the wind stress.



d) The total volume transport:

The volume transport is always 90o to the direction of the wind stress. On the northern hemisphere, it is on the right of the wind stress.

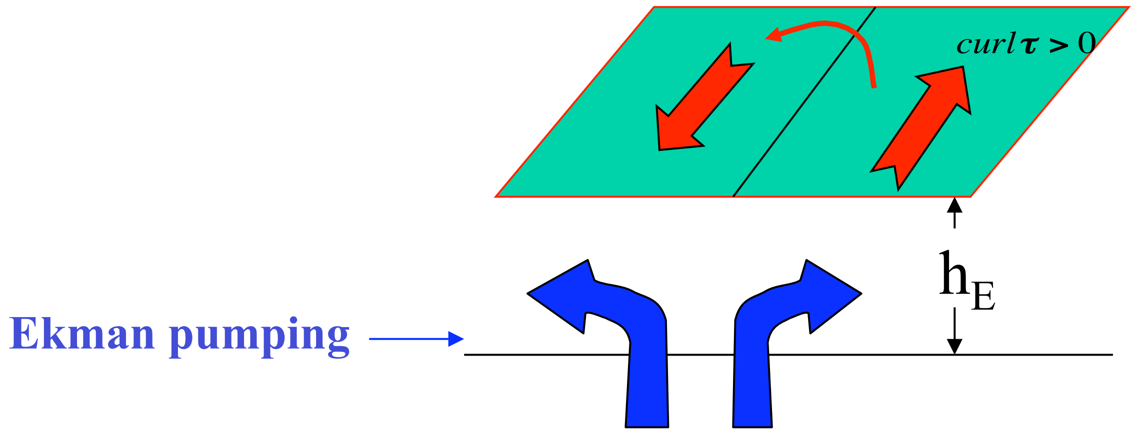
Ekman Pumping:

$$\int_{-h_E}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dz = 0 \quad \Rightarrow w\big|_{z=-h_E} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \frac{1}{\rho f}\boxed{\frac{\partial \tau_y}{\partial x}}$$

If we extend it to a general case with x and y components of the wind, we get

$$\boldsymbol{w}\big|_{z=-h_E} = \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{y}} = \frac{1}{\rho \boldsymbol{f}}\left(\frac{\partial \tau_y}{\partial \boldsymbol{x}} - \frac{\partial \tau_x}{\partial \boldsymbol{y}}\right) = \boxed{\frac{1}{\rho_o \boldsymbol{f}} \boldsymbol{k} \cdot curl\, \boldsymbol{\tau}}$$

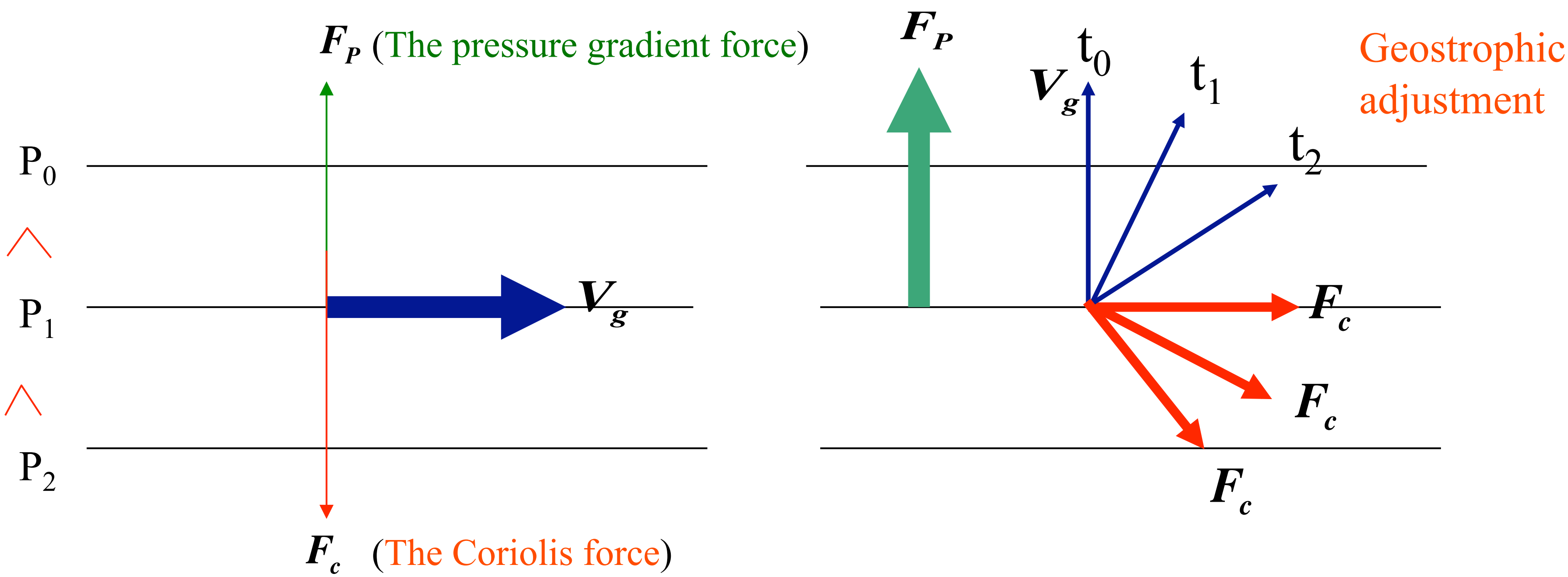
The vertical velocity at the bottom of the surface Ekman layer is proportional to the vorticity of the surface wind stress and it is independent of the detailed structure of the Ekman flow. the vertical velocity is positive for the positive vorticity of the wind stress, and negative for the negative vorticity of the wind stress.



2) Interior geostrophic layer

Turbulent mixing is weak, the motion is approximately geostrophic

$$\boxed{u_g = \frac{1}{\rho f}\frac{\partial P}{\partial y}, \quad v_g = -\frac{1}{\rho f}\frac{\partial P}{\partial x}} \longrightarrow \boxed{\text{Coriolis force = Pressure gradient force}}$$



Where does the energy come here? $\rightarrow$ Vorticity of the wind stress (Ekman pumping)

3) The Bottom Ekman boundary layer

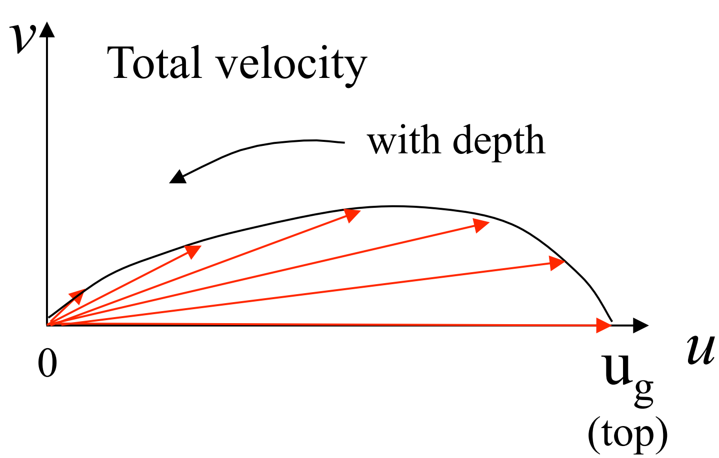
$$\begin{cases} -fv = -\dfrac{1}{\rho}\dfrac{\partial P}{\partial x} + K_m \dfrac{\partial^2 u}{\partial z^2} \\ fu = -\dfrac{1}{\rho}\dfrac{\partial P}{\partial y} + K_m \dfrac{\partial^2 v}{\partial z^2} \end{cases}$$

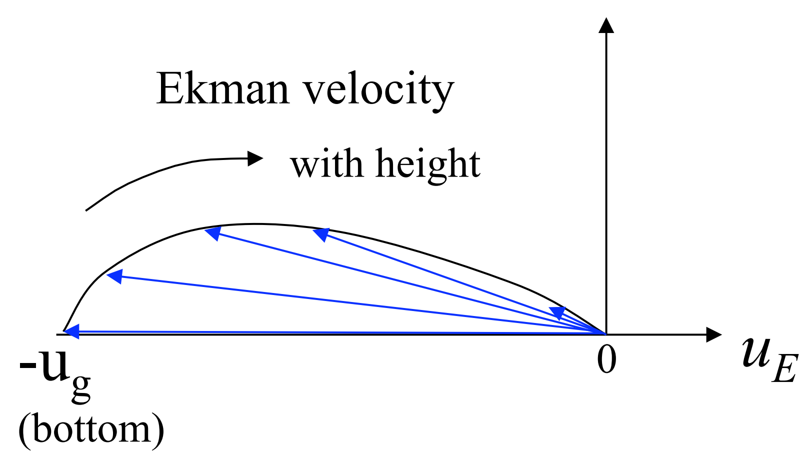
Coriolis force=Pressure gradient force+Vertical diffusion

Surface wind stress drives the surface Ekman layer and also transport the energy the interior layer. The energy driving the bottom boundary layer could be come from the interior geostrophic motion.

a) <u>Total velocity and Ekman velocity</u>:

Total velocity (a sum of geostrophic and Ekman velocities) rotates counterclockwise with depth. It reaches the geostrophic current at the top of the Ekman layer and vanishes at the bottom. Ekman velocity decreases and rotates clockwise with the height (from the bottom. It is equal in magnitude and opposite in direction to the geostrophic velocity at the bottom and vanish at the top of the bottom Ekman layer.



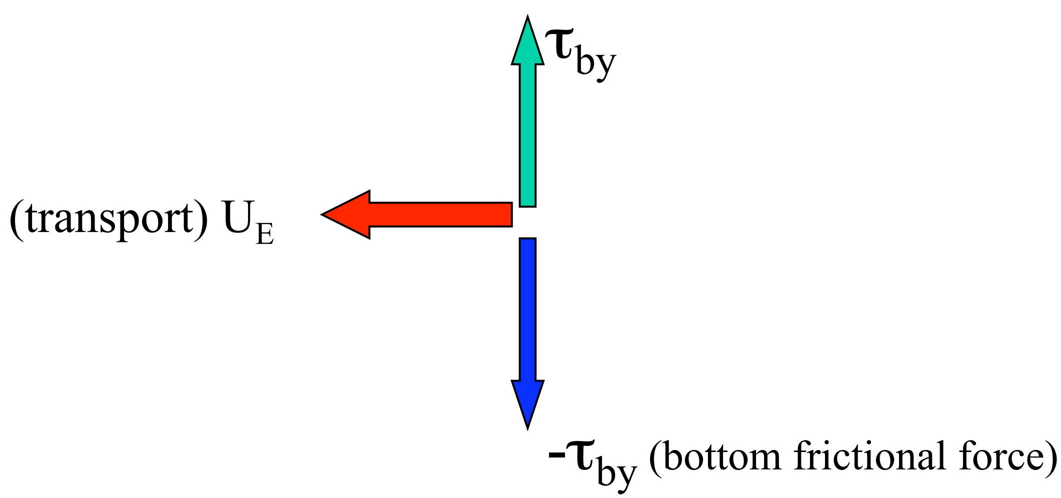


d) Total volume transport:

The volume transport is always 90° on the right of the bottom frictional force (on the northern hemisphere). Unlike the volume transport in the surface Ekman layer, the transport in the bottom boundary is related to the Ekman layer thickness and magnitude of the geostrophic current in the interior.

$$\boxed{U_E = -\frac{\tau_{by}}{f} = -\frac{K_m}{fh_E}u_g = -\frac{h_E^2}{2h_E}u_g = -\frac{1}{2}u_g h_E}$$

Note: τ_{by} is the bottom stress exerting from the fluid on the bottom. The bottom frictional force is equal in magnitude and opposite to the bottom stress.



4) The bottom log boundary layer

In a very thin layer close to the bottom, the turbulent viscosity-induced stress is much larger than the Coriolis and pressure gradient forces. In this layer, the stress does not change with depth:

$$\boxed{\frac{\partial \boldsymbol{\tau}}{\partial z} = 0}$$

→ **This is also called the constant stress boundary layer**

According to the definition of the stress, we have

$$\tau = \rho K_m \frac{\partial u}{\partial z} \quad \Rightarrow \frac{\partial u}{\partial z} = \frac{1}{K_m}\left(\frac{\tau}{\rho}\right) = \frac{u_*^2}{K_m}$$

u_* : the frictional velocity

Assume that mixing length $l = \kappa z$, where κ is von Karman constant , then we have

$$\frac{\partial u}{\partial z} = \frac{u_*}{l} = \frac{u_*}{\kappa z} \quad \Rightarrow u = \frac{u_*}{\kappa}(\ln z + C)$$

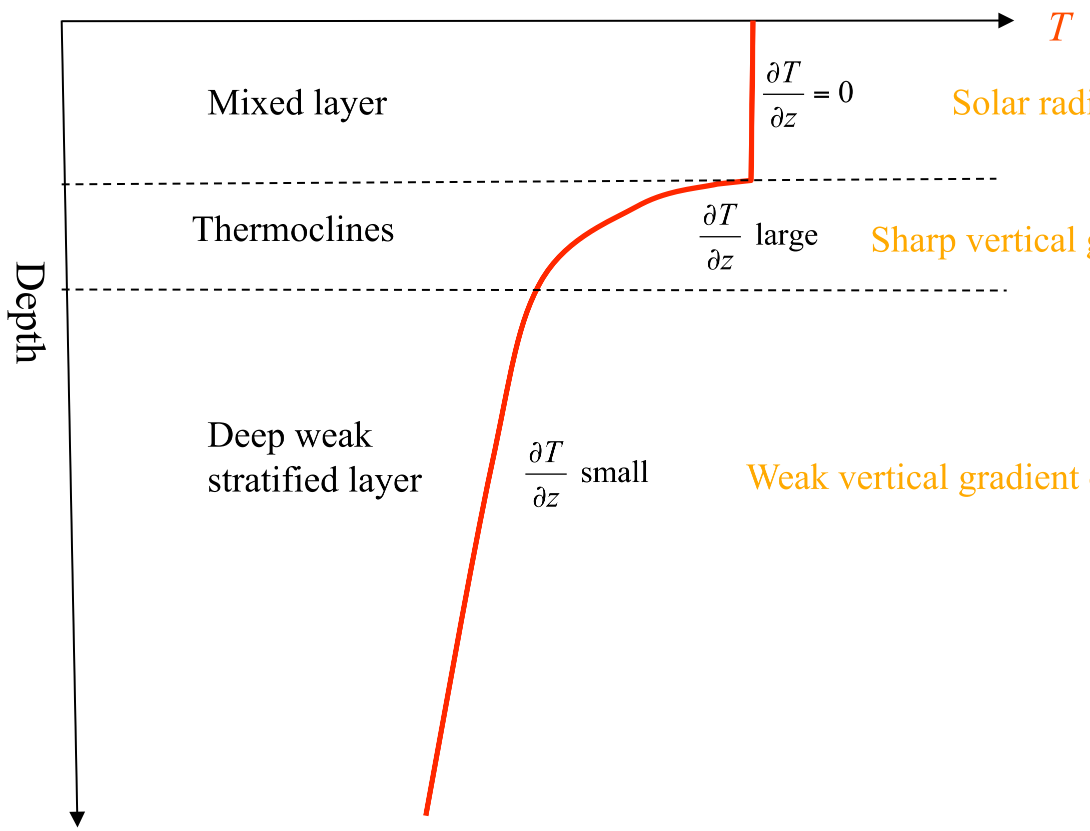
Define z_o is the roughness height, at which turbulence-induced stress equals to zero, then

$$\boxed{u = \frac{u_*}{\kappa} \ln \frac{z}{z_o}}$$

z_o : 0.01 mm ~ 60 mm

Surface Mixed Layer

General features of vertical structure of the water temperature:



Solar radiation, wind mixing/cooling

Sharp vertical gradient of the water temperature

Weak vertical gradient of the water temperature

Question 1: Why do the thermoclines exist in the ocean?

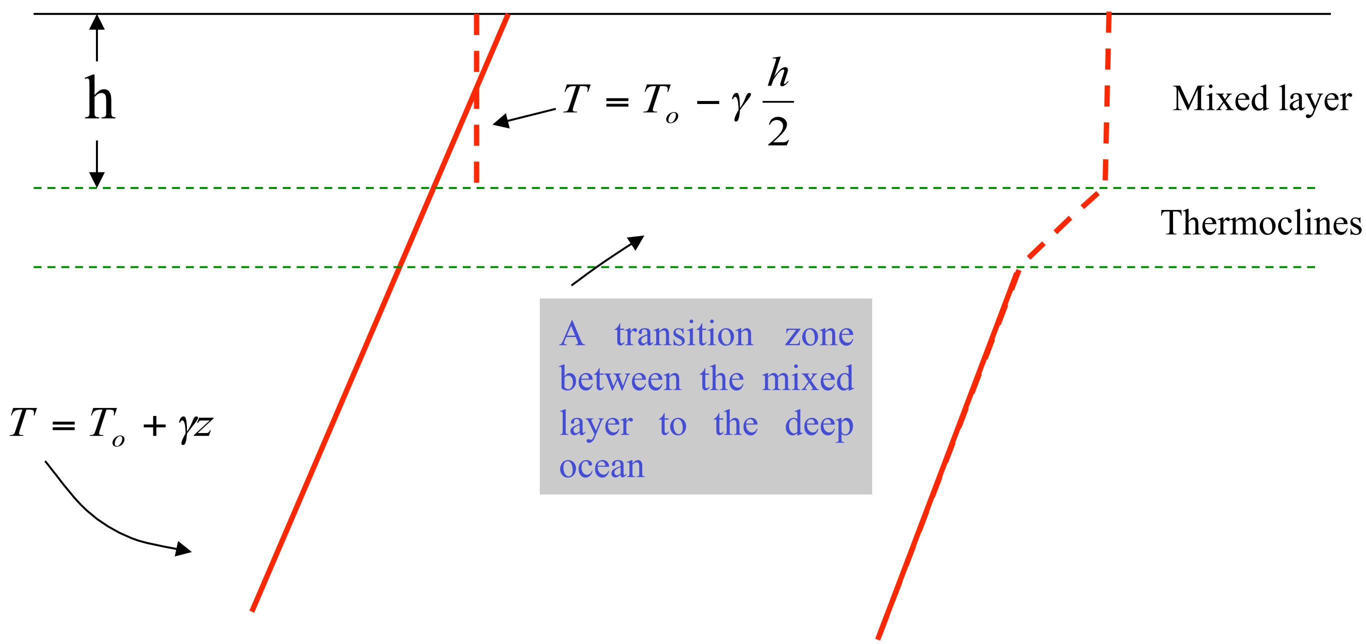
An example:

Assume that solar radiation leads to a vertically linear distribution of the water temperate:

$$T = T_o + \gamma z$$

Wind mixing and cooling cause a vertical mixing, resulting in a mixed layer of h and a temperature as

$$\overline{T} = \frac{1}{h}\int_{-h}^{0}(T_o + \gamma z)dz = T_o - \gamma\frac{h}{2}$$ An averaged temperature from 0 to -h



For density: pycnoclines; for salinity: thermohalines

Question 2: Is the oceanic current is a turbulent motion?

Reynold number:

$$\boxed{R_e = \frac{VL}{\gamma}}$$

V: velocity of the motion
L: typical scale of the motion
γ: fluid's viscosity

Laboratory experiment: R_e > 2000 or 4000 (depending on the property of the fluid), the motion become turbulent!

In the ocean,

$R_e > 10^{11}$ (for example, in the Gulf Stream, $V \sim 1$ m/s, $L \sim 100$ km, and $\gamma \sim 10^{-6}$ m^2/s)

The oceanic current is a turbulent motion!

Question 3: What causes turbulence and how is it dissipated?

Shear Instability

↓

Large-scale eddies

↓

Small-scale eddies

↓

Dissipation

Assume that

u^2: the typical scale of the kinetic energy of the large-scale eddies;

l: the typical size scale of the large-scale eddies;

The time scale required for the energy transfer of large-scale eddies to small-scale eddies is

$$\frac{l}{u}$$

The energy transfer rate from large-scale eddies to small-scale eddies equals:

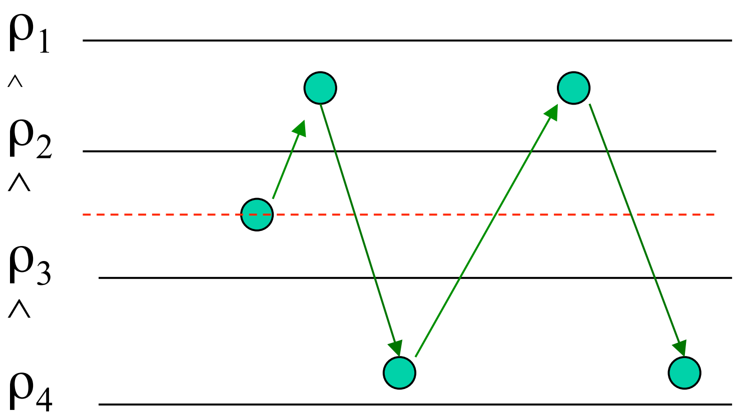
$$\frac{\text{turbulent kinetic energy}}{\text{time scale}} \sim \frac{u^2}{l/u} = \frac{u^3}{l}$$

If these small-scale eddy's energy is dissipated at a rate of ε, then

Turbulent dissipation rate ←

$$\boxed{\varepsilon \sim \frac{u^3}{l}}$$

Question 4: How does shear instability cause turbulence?



Moving a blob of water in a stable density gradient leads to vertical oscillations.

The buoyancy force defined as its "weight" equals to:

$$F_g = -g(\rho_b - \rho_s)$$

ρ_b is the density of the blob; ρ_s is the density of the surrounding water

The frequency of the blob's oscillation is related to how fast it changes direction, It is controlled by the buoyancy force. In fact, the frequency of the oscillation is equal to

$$\boxed{N = \sqrt{-\frac{g}{\rho_o}\frac{\partial \rho}{\partial z}}}$$

Brunt-Väisälä frequency

Internal wave frequency:

$$\boxed{f \leq \omega \leq N}$$

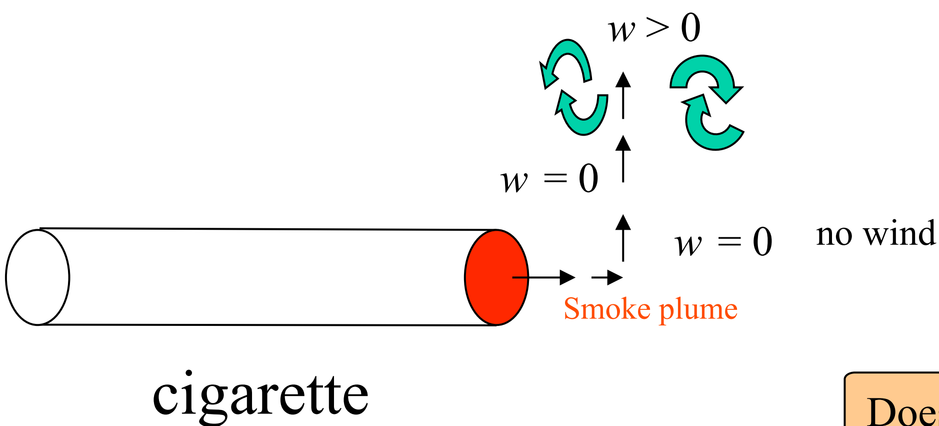
How could turbulence mixing occur in a stable stratified fluid?

One of the most important forces to cause vertical mixing is the vertical shear of the horizontal velocity, i.e,:

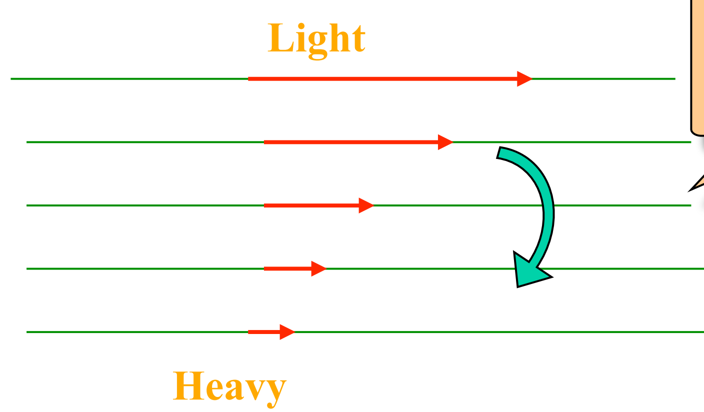
$$\boxed{\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}}$$

Why do the shear of the velocity is so important for mixing?

Example:



In the ocean:



Does any shear of the velocity generate turbulence?

Richardson number

The occurrence of the turbulence mixing is dampened by strong density gradient, but strengthened by strong shear of the velocity. Whether or not turbulence mixing could occur depends on a ratio of vertical stratification to the vertical shear of the velocity.

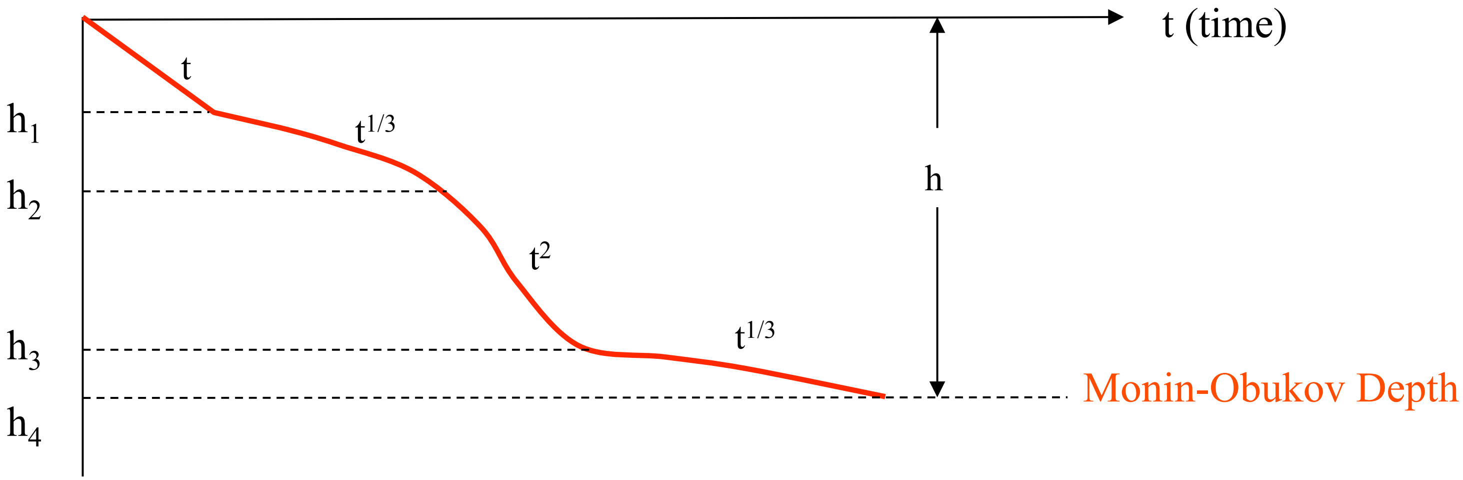
Richardson number is defined as a ratio of the square of the buoyancy frequency to the square of the vertical shear of the horizontal velocity:

$$\boxed{R_i = N^2 / \left(\frac{\partial u}{\partial z} \right)^2}$$

In general, turbulence develops as $R_i < 0.25$, but recent microstructure measurement shows that critical Richardson number is bigger for the interior ocean mixing or over the slope.

Question 5: How is a mixed layer developed? What are the key balance of turbulence energy during the development of a mixed layer?

A mixed layer is developed with 4 stages:



Stage 1: $t << 1/f$ (time scale << the inertial period), wind stress is limited in a thin layer close to the surface, $h \sim t$

Stage 2: the mixed layer continues to deepen, but cooler water below the mixed layer is entrained, the deepening speed of the mixed layer decreases: $h \sim t^{1/3}$

Stage 3: a strong vertical shear of the velocity is established, shear instability speed up mixing. In this stage, $h \sim t^2$

Stage 4: Energy generated by the vertical shear of the velocity is balanced by the heat-induced buoyancy force, the mixed layer stops deepening. This is a slowly process during which $h \sim t^{1/3}$.

Monin-Obukov Depth: $\boxed{\tilde{h} = \frac{2\rho_o m_o u_*^3}{g\alpha Q}}$

$u_* = \sqrt{|\tau_o|/f}$: frictional velocity; ρ_o: reference density

τ_o: surface wind stress, Q: the surface heat flux,

m_o: turbulent dissipation coefficient

The most important properties of a mixed layer is

$$\boxed{\frac{\partial \boldsymbol{V}}{\partial z} = 0; \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial S}{\partial z} = 0, \quad \frac{\partial \rho}{\partial z} = 0}$$

Question 6: For a given same wind stress, the wind could drive an Ekman flow, but also wind mixing leads to a mixed layer. Based on the Ekman theory, the wind-induced current has a maximum speed at the surface and decreases exponentially with depth. According to the theory of the mixed layer, no vertical gradient of the velocity exists in a mixed layer. Are these two theories controversial each other?

Answer: No! When mixing occurs, the turbulent viscosity coefficient is very large, turbulent mixing tends to mix density, temperature and salinity, but also tends to mix the momentum. Therefore, the momentum of the Ekman current will be "mix" in the vertical.

In the mathematics, it is equivalent to

$$\begin{cases} \dfrac{\partial u_E}{\partial z} = -\dfrac{2\tau_y}{\rho_o f h_E^2} e^{\frac{z}{h_E}} \sin\dfrac{z}{h_E} \approx -\dfrac{2\tau_y}{\rho_o f h_E^2}[1 + \dfrac{z}{h_E} + O(\dfrac{z^2}{h_E^2})][\dfrac{z}{h_E} - O(\dfrac{z^2}{h_E^2})] \\ \dfrac{\partial v_E}{\partial z} = \dfrac{2\tau_y}{\rho_o f h_E^2} e^{\frac{z}{h_E}} \cos\dfrac{z}{h_E} \approx \dfrac{2\tau_y}{\rho_o f h_E^2}[1 + \dfrac{z}{h_E} + O(\dfrac{z^2}{h_E^2})][\dfrac{z}{h_E} - O(\dfrac{z^2}{h_E^2})] \end{cases}$$

As $h_E \rightarrow \infty$ $$\boxed{\frac{\partial u_E}{\partial z} = \frac{\partial v_E}{\partial z} = 0}$$

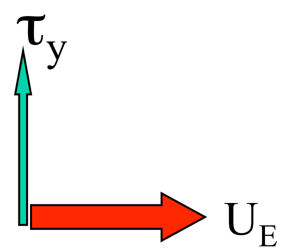
Question 7: Is the Ekman theory still valid when vertical mixing occurs?

Answer: Yes.

Why? Mixing tends to re-distribute the momentum but does not change the transport!

The wind-induced Ekman transport:

$$U_E = \frac{\tau_y}{\rho_o f}$$



Ekman transport

The wind-induced Ekman transport depends only on the wind stress and Coriolis parameter rather than the detailed structure of the Ekman current. Therefore, wind mixing tends to cause the Ekman current to be uniform in the vertical in the mixed layer, but would not be able to change the Ekman transport!

The Ekman theory is still valid in the mixed layer!

Mixed Layer Models in the Coastal Ocean

Two types of the mixed layer model:

1) PWP (Price et al., 1986: Price, Weller and Pickel):
Mixing is determined based on the criterions of the turbulent motion:

a) static instability; b) mixed layer instability, and c) shear-flow instability

2) Mellor and Yamada 2.5 turbulent closure (Mellor and Yamada, 1974, 1982):
Mixing is determined by the turbulent kinetics and mixing length equations—a diffusion process:

a) Source: velocity shear and buoyancy instability
b) Re-distribution: turbulent advection and diffusion
c) Sink: turbulent dissipation

PWP is 1-D mixing model which is popular in the open ocean ecosystem studies because it is very simple.

MY2.5 is a 3-D mixing model which is widely used in the coastal ocean.

PWP Mixed Layer Model:

Wind stress + Heat flux

Static instability

$$\frac{\partial \rho}{\partial z} > 0$$

Mixed layer instability

$$R_{ib} = \frac{g \Delta \rho h}{\rho_o (\Delta u)^2} < 0.65$$

Shear instability

$$R_{ig} = \frac{N^2}{(\partial u / \partial z)^2} < 0.25$$

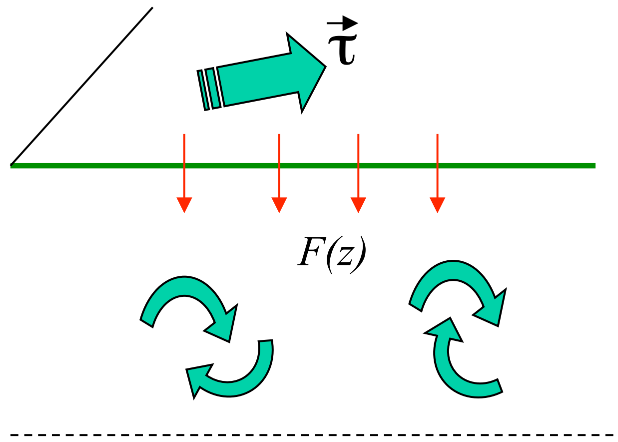
Re-distributions of T, S, ρ, U

1) Mixed layer depth
2) Vertical profile of the horizontal velocity;
3) Vertical profile of temperature, salinity, and density

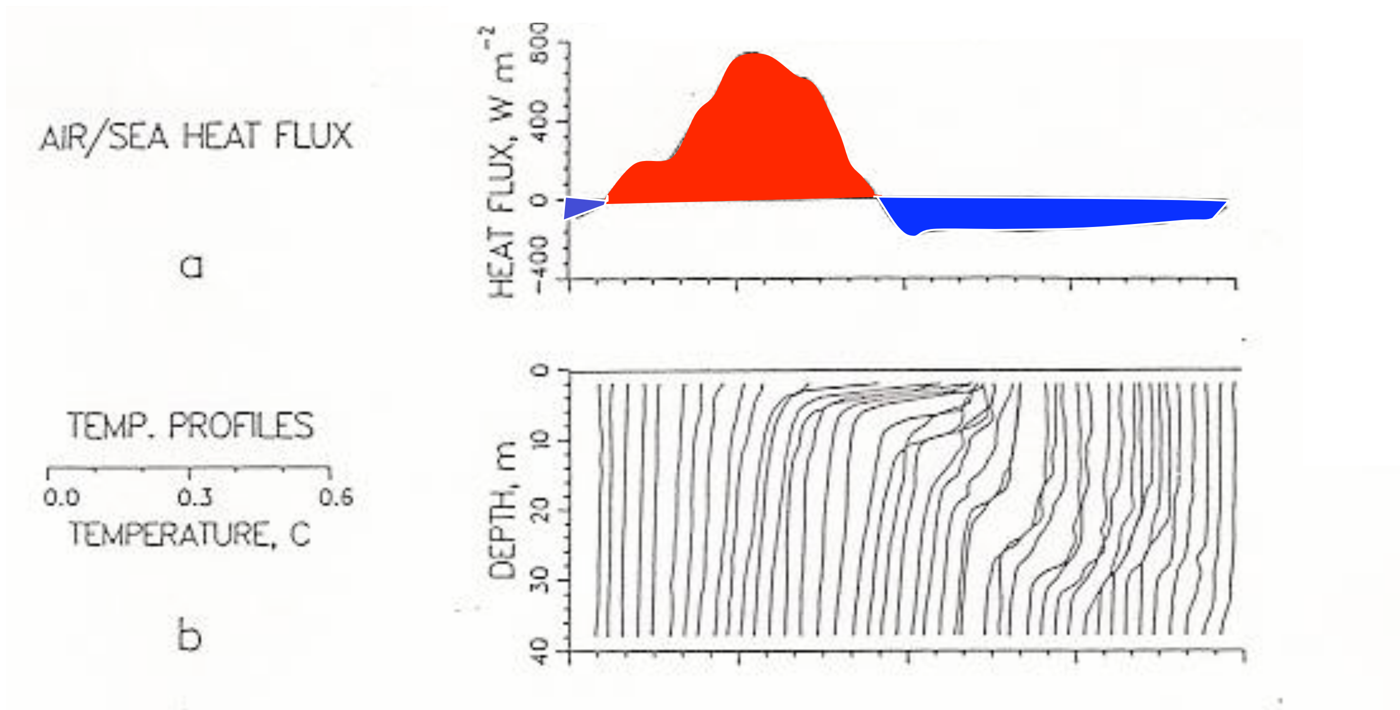
Δρ and Δu are the differences of density and velocity at the bottom of the mixed layer between the mixed and stratified layers.

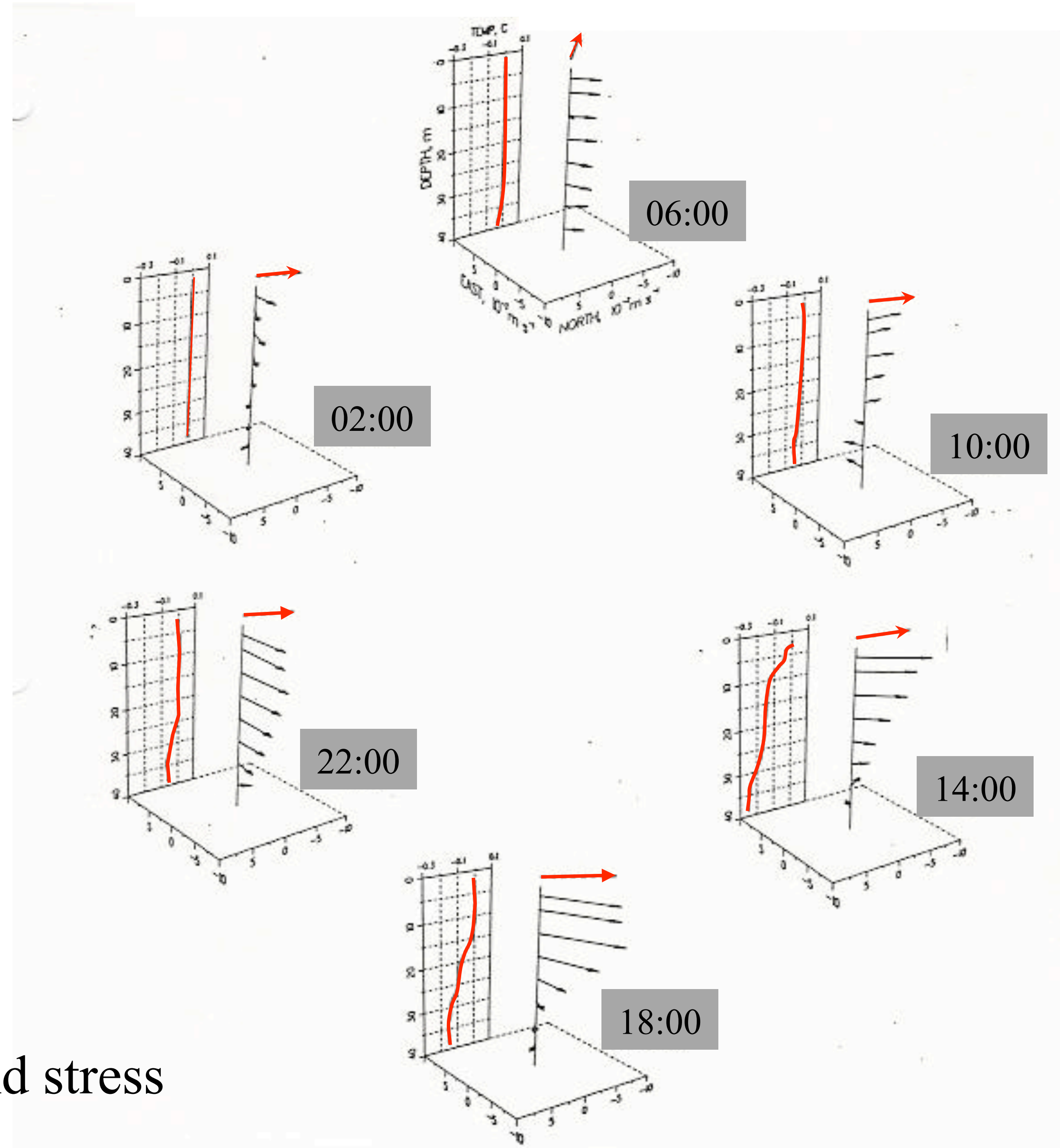
An example: 1-D mixing experiment

$$\begin{cases} \dfrac{\partial T}{\partial t} = -\dfrac{1}{\rho_o c_p}\dfrac{\partial F}{\partial z} \\ \dfrac{\partial u}{\partial t} = fv - \dfrac{1}{\rho_o}\dfrac{\partial \tau_x}{\partial z} \\ \dfrac{\partial v}{\partial t} = -fu - \dfrac{1}{\rho_o}\dfrac{\partial \tau_y}{\partial z} \end{cases}$$

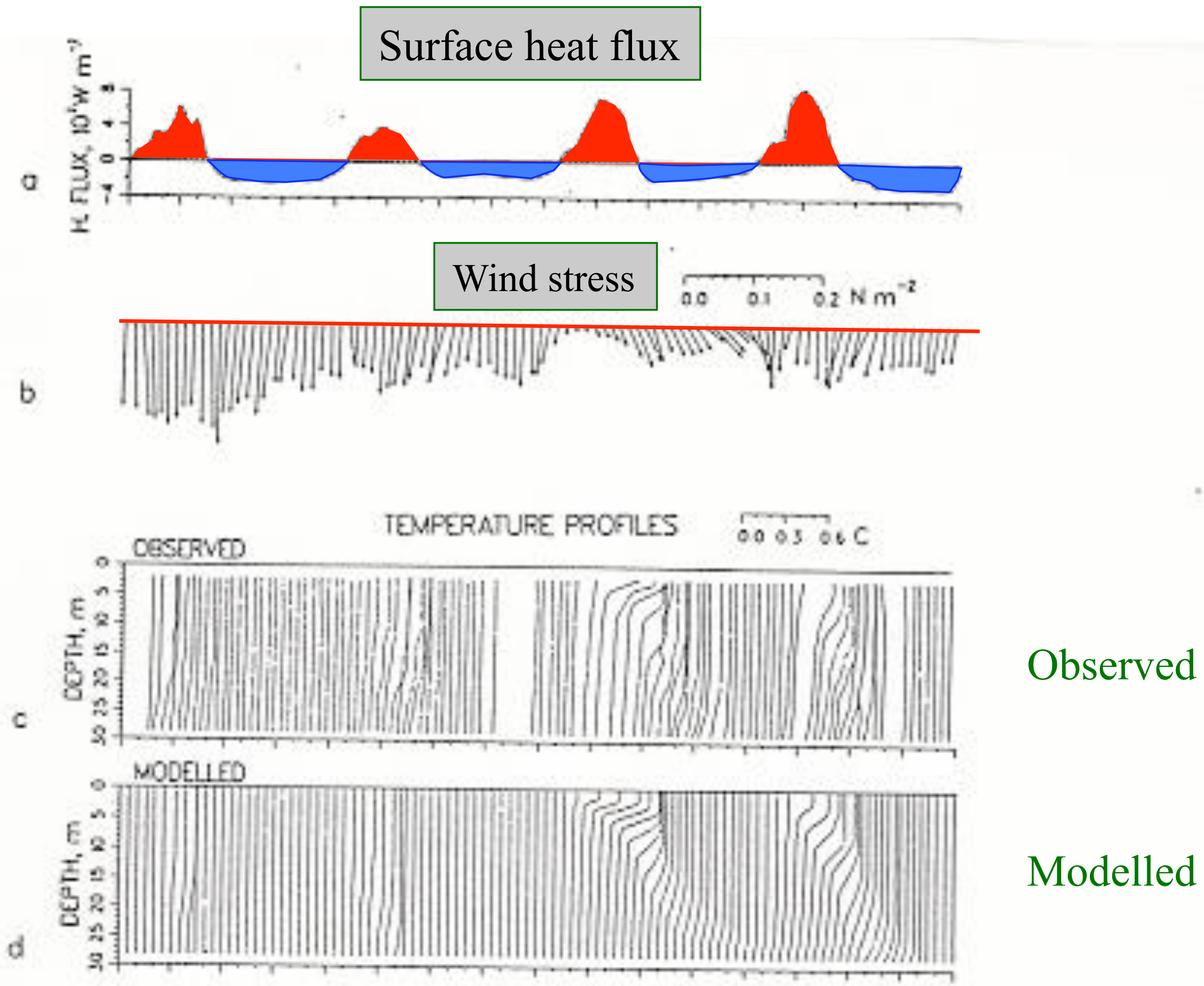


F(z) is the short-wave isolation. In this experiment, it changes with diurnal cycle: a diurnal heating

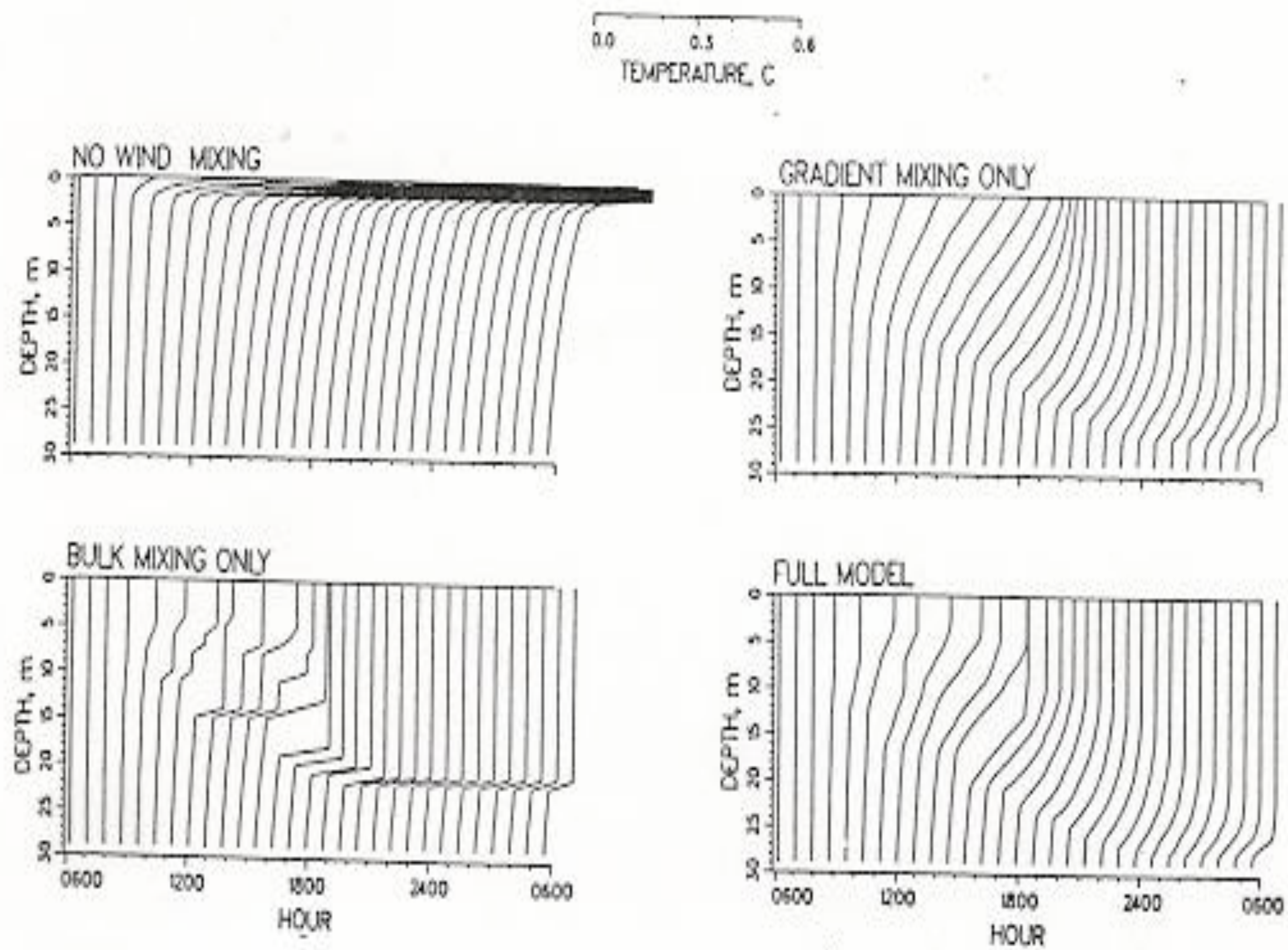



TEMP, C
DEPTH, m
EAST, 10^{-2} m s^{-1}
NORTH, 10^{-2} m s^{-1}
06:00
02:00
10:00
22:00
14:00
18:00

wind stress



Surface heat flux
H. FLUX, 10²W m⁻²
a
Wind stress
0.0 0.1 0.2 N m⁻²
b
TEMPERATURE PROFILES
0.0 0.3 0.6 C
OBSERVED
DEPTH, m
c
MODELLED
DEPTH, m
d
Observed
Modelled


0.0
0.3
0.6
TEMPERATURE, C
NO WIND MIXING
GRADIENT MIXING ONLY
BULK MIXING ONLY
FULL MODEL
DEPTH, m
HOUR
0600
1200
1800
2400
0600

MY Turbulent Closure Model

1) Level 2.0 closure model

Turbulent production = Turbulent energy dissipation

$$P_s = K_m[(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2] \longrightarrow$$ turbulent shear production

$$P_b = K_h(\frac{g}{\rho}\frac{\partial \rho}{\partial z}) \longrightarrow$$ turbulent buoyancy production

$$\varepsilon = 0.06q^3/l \longrightarrow$$ turbulent dissipation

where $q^2 = (u'^2 + v'^2)/2$, $l = l_{\max}\frac{\kappa z}{1+\kappa z}$

Therefore,

$$\boxed{K_m = lqS_m, \quad K_h = lqS_h}$$

$$\begin{cases} S_h = 0.537 - 1.978R_j/(1-R_j) \\ S_m = S_h \dfrac{0.52 - 1.404R_j/(1-R_j)}{0.688 - 2.068R_j/(1-R_j)} \end{cases}$$

$$R_j = 0.725[(R_i + 0.186 - (R_i^2 - 0.316R_i + 0.0346)^{1/2}]$$ (Blackadar, 1962, Weatherly and Martin, 1978)

2) Level 2.5 closure model

$$\frac{\partial q^2}{\partial t} + u\frac{\partial q^2}{\partial x} + v\frac{\partial q^2}{\partial y} + w\frac{\partial q^2}{\partial z} = 2(P_s + P_b - \varepsilon) + \frac{\partial}{\partial z}(K_q \frac{\partial q^2}{\partial z}) + F_q$$

$$\frac{\partial q^2 l}{\partial t} + u\frac{\partial q^2 l}{\partial x} + v\frac{\partial q^2 l}{\partial y} + w\frac{\partial q^2 l}{\partial z} = lE_1(P_s + P_b - \frac{\widetilde{W}}{E_1}\varepsilon) + \frac{\partial}{\partial z}(K_q \frac{\partial q^2 l}{\partial z}) + F_{q^2 l}$$

and

$$K_m = lqS_m, \quad K_h = lqS_h, \quad K_q = 0.2lq$$

where

$$S_m = \frac{0.4275 - 3.354G_h}{(1 - 34.676G_h)(1 - 6.127G_h)}, \quad S_h = \frac{0.494}{1 - 34.676G_h}$$

$$G_h = \frac{l^2 g}{q^2 \rho_o}\frac{\partial \rho}{\partial z}$$

A Simple Mixed Layer Ecosystem Model

For a linear case,

$$\frac{\partial \boldsymbol{v}}{\partial t} + f\,\boldsymbol{k} \times \boldsymbol{v} = \frac{1}{\rho_o}\frac{\partial \boldsymbol{\tau}}{\partial z} \qquad (1)$$

In the mixed layer,

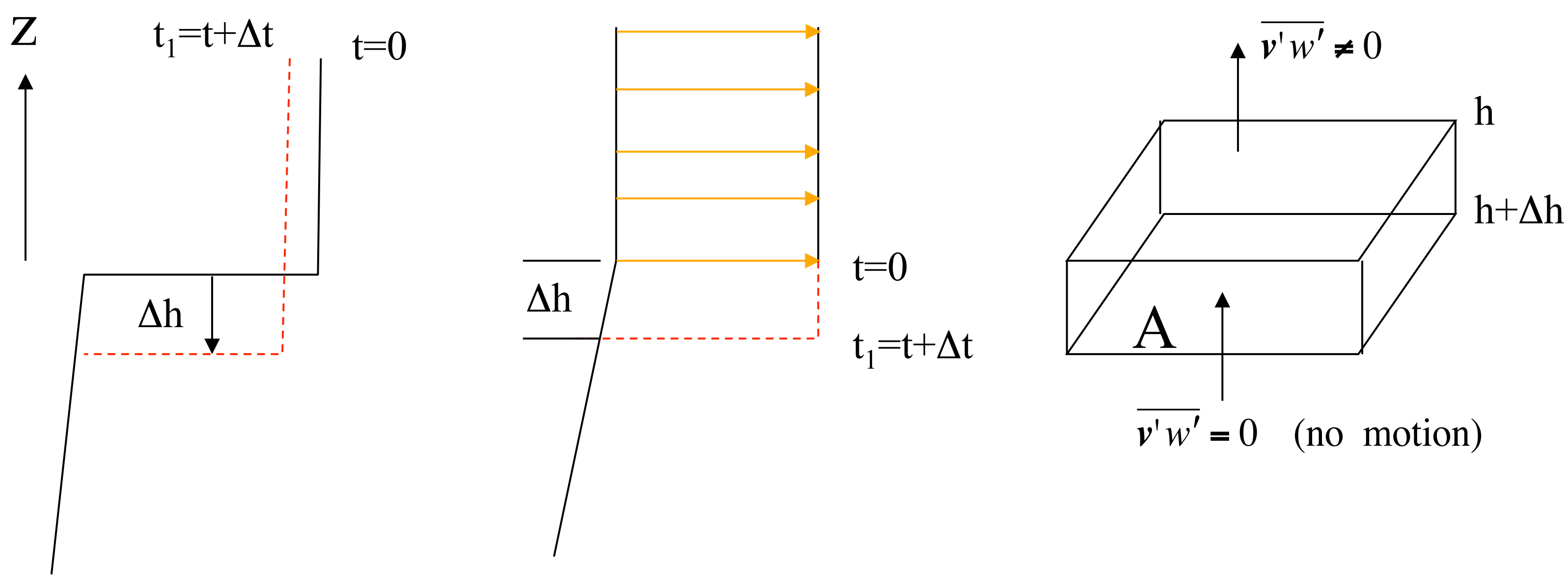
$$\boxed{\frac{\partial^2 \boldsymbol{\tau}}{\partial z^2} = 0} \qquad (2)$$

Integrating (1) from $-h$ to 0

$$h\frac{\partial \boldsymbol{v}}{\partial t} + f\,\boldsymbol{k} \times \boldsymbol{v}\,h = \boldsymbol{\tau}\big|_{z=0} - \boldsymbol{\tau}\big|_{z=-h} = \boldsymbol{\tau}_s - \boldsymbol{\tau}\big|_{z=-h}$$

The stress at the bottom of the mixed layer is related to *h* !

Question: How to determine the stress at the bottom of the mixed layer?



Stress at –h:

$$\tau\big|_{z=-h} = -\rho\overline{v'w'}$$

When the mixed layer deepens from h to h+ Δh over a time interval Δt, water below the mixed layer intrudes into the mixed layer to participate in mixing, the change rate of the total momentum equals to the difference of the turbulent momentum flux at the h and h+ Δh surfaces, i.e.,

$$\frac{d(\rho v\Delta h A)}{dt} = \rho A(-\overline{v'w'}\big|_{z=-h} + \cancelto{0}{\overline{v'w'}\big|_{z=-h-\Delta h}})$$

As $\Delta h \rightarrow 0$, we have

$$\boxed{\boldsymbol{\tau}\big|_{z=-h} = -\rho \overline{\boldsymbol{v}' w'}\big|_{z=-h} = \rho \boldsymbol{v} \frac{\partial h}{\partial t}}$$

deeper

When the mixed layer becomes shallower, no deep water's intrusion, so

$$\frac{d(\rho \boldsymbol{v} \Delta hA)}{dt} = 0; \quad \boldsymbol{\tau}\big|_{z=-h} = -\rho \overline{\boldsymbol{v}' w'}\big|_{z=-h} = 0$$

shallower

Therefore,

$$\boxed{h \frac{\partial \boldsymbol{v}}{\partial t} + f \boldsymbol{k} \times \boldsymbol{v} h = \frac{\boldsymbol{\tau}_s}{\rho} - \boldsymbol{v} \frac{\partial h}{\partial t} H(\frac{\partial h}{\partial t})}$$

$$\begin{aligned} H(\xi) &> 1 \quad \text{as } \xi > 0 \\ &= 0 \quad \text{as } \xi \le 0 \end{aligned}$$

For example, $\tau_x=0$, and $\tau_y=\tau_o$ (constant)

$$\frac{\partial uh}{\partial t} - fvh = 0 \quad , \frac{\partial vh}{\partial t} + fuh = -\frac{\tau_o}{\rho_o} \qquad (3)$$

The solution is

$$\boxed{\begin{cases} uh = \dfrac{\tau_o}{\rho f}(\cos ft - 1) \\ vh = -\dfrac{\tau_o}{\rho f}\sin ft \end{cases}}$$

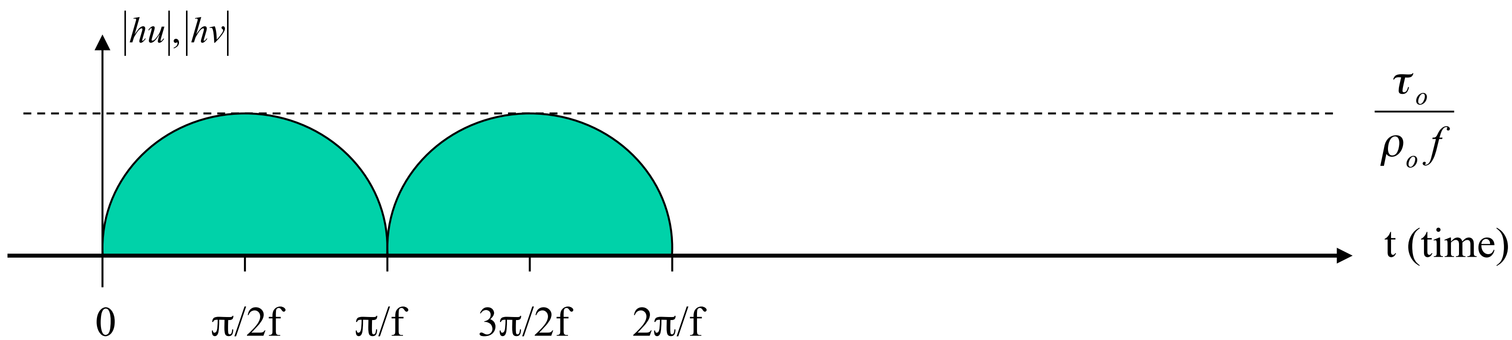
Inertial oscillations

Ekman transport

Properties:

1) Velocity in the mixed layer decreases as the mixed layer depth increases, but the total transport is determined by the Ekman transport which is independent of the velocity in the mixed layer,

2) The motion consists of two parts: 1) time-dependent inertial oscillations and 2) wind-induced steady Ekman transport. The period of the oscillation is 2π/f.



Consider the nutrient transport in the mixed layer: we have

$$h\frac{\partial N}{\partial t} + uh\frac{\partial N}{\partial x} + vh\frac{\partial N}{\partial y} + (N - N_o)\frac{\partial h}{\partial t}H(\frac{\partial h}{\partial t}) = Q_N$$

where Q_N is the nutrient flux at the surface, N_o is the nutrient concentration in the stratified layer below the mixed layer

1) Case 1: $Q_N = 0, \frac{\partial N}{\partial x} = \frac{\partial N}{\partial y} = 0, \frac{\partial h}{\partial t} > 0$,we have

$$h\frac{\partial N}{\partial t} + (N - N_o)\frac{\partial h}{\partial t} = 0$$

If $N_o > N$, then

$$\boxed{\frac{\partial N}{\partial t} = (N - N_o)\frac{\partial}{\partial t}(\frac{h^2}{2}) > 0}$$

When the mixed layer deepens, the deeper nutrients will entrain the mixed layer to increase the nutrient concentration in the mixed layer.

When N_o is a constant, we have

$$h(N - N_o) = \text{constant}$$

Let $N = N_1$ at $h = h_1$, then

$$hN = h_1 N_1 + (h - h_1) N_o$$

2) Case 2: $Q_N = 0, \quad N_o = 0 \quad \text{and } \partial h/\partial t = 0$

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = 0 \longrightarrow \frac{dN}{dt} = 0$$

Assume that

$$N = \overline{N}(t)e^{-\lambda(x+y)}$$

Yield:

$$\frac{\partial N}{\partial t} - \lambda(u + v)\overline{N} = 0 \Rightarrow \frac{\partial \ln \overline{N}}{\partial t} = \frac{\lambda \tau_o}{h\rho f}(\cos ft - \sin ft - 1)$$

Solution:

$$N = Ce^{\frac{\lambda\tau_o}{h\rho f}t} e^{\frac{\lambda\tau_o}{h\rho f^2}(\sin ft + \cos ft)} e^{-\lambda(x+y)}$$

At a local position, the high nutrient center will decrease exponentially with time in the mixed layer. The time scale required to mix all the nutrients is

$$\boxed{T_N \sim \frac{h}{\lambda(\tau_o / \rho f)}}$$

Since in this case the nutrient is conservative when it is advected, the decrease of nutrients at a local position is resulted from the advective process due to the Ekman transport.

Note: one probably could find that N **varies locally at inertial frequency.**